# Fair inheritance taxation*

Benoit Decerf[†] Francois Maniquet[‡]

September 8, 2021

**Abstract**

We study the optimal taxation of bequests in a version of the model of Piketty and Saez (2013). Agents have heterogeneous preferences over their consumption and the net-of-tax bequest received by their heir. The bequest left by an individual depends on both her degree of altruism and the bequest received from her parents. First, we study two principles at the heart of the debates on taxing inheritances: 1) children should not be penalized by the lack of altruism of their parents; 2) parents should be free to choose their bequests. Only one social welfare function (SWF) satisfies these two principles, together with Pareto efficiency and a separability principle. Second, we study the shape of the inheritance tax scheme that maximizes this SWF. We show that, in the aggregate, the inheritance tax must collect money (redistributed through a non-negative demogrant). Moreover, small bequests cannot be taxed (they can potentially be subsidized), while bequests larger than that of the most altruistic individuals who did not receive bequests from their parents should be taxed as much as efficiency permits.

Keywords: fairness, inheritance taxation, responsibility, compensation, tax exemption

JEL: D63, D64, H21

*Acknowledgments : We are grateful to Dirk Neumann and Kevin Spiritus for helpful discussions. We thank all the participants to the DEFIPP workshop of March 2017 (University of Namur), the "What is socialism today" conference (Yale University), for their comments. Funding from the Banque Nationale de Belgique (Exercice 2017) is gratefully acknowledged. The findings, interpretations, and conclusions expressed in this paper are entirely those of the authors and should not be attributed in any manner to the World Bank, to its affiliated organizations, or to members of its Board of Executive Directors or the countries they represent. The World Bank does not guarantee the accuracy of the data included in this publication and accepts no responsibility for any consequence of their use. The usual disclaimer applies.

[†]World Bank. bdecerf@worldbank.org

[‡]University of Louvain-la-Neuve. francois.maniquet@uclouvain.be

# 1 Introduction

Piketty and Saez (2013) deeply questioned the social desirability of recent reforms in the inheritance tax systems. Indeed, the outcome of these reforms is a decrease in inheritance tax, whereas Piketty and Saez (2013) showed that the historical high top inheritance tax rates near 70% observed in the US and the UK over the 50's, 60's and 70's were within the brackets of the optimal ones (see Figure 3 in Piketty and Saez (2013)). Optimality here is measured with respect to the maximization of some social welfare function.

In this paper, we solve two problems that are left unsolved in Piketty and Saez (2013). First, we show how fairness principles can be used to endogenize social welfare weights. In Piketty and Saez (2013), indeed, the formula of the optimal tax is a function of how society values the utility of different individuals, but in a world in which individuals differ in many dimensions, all of which likely to create inequality, it is not clear who should be given priority.[1] We solve this problem by resorting to two fairness principles, which are at the heart of the debates on taxing inheritance. The first principle is that parents should be free to choose their bequest (McCaffery, 1994). This is consistent with the principle of responsibility for one's preferences, which has inspired recent developments in optimal taxation theory (like, among others, Fleurbaey and Maniquet (2018), Lockwood and Weinzierl (2016), Piketty and Saez (2013)). The second fairness principle is that children should not be penalized by the lack of altruism of their parents.

These two fairness principles may sound in conflict, but we show that they are not. This comes from the fact that each individual is both a child, who may wish to receive a part of the bequest allocated to children of other families, and a parent, who wishes to be free to bequeath the amount she prefers. As a result, there exists one social welfare function that reconciles both principles. This is the social welfare function that we maximize to identify the shape of the optimal tax function.

Second, and most importantly, we identify the shape of the optimal inheritance tax on low bequests, that is bequests left by parents who did not receive anything from their own parents. In Piketty and Saez (2013), indeed, the tax is assumed to be either linear, or linear after an interval of exemption. Starting with a much broader set of admissible tax functions, we prove that the optimal function has one of the following two shapes. Either it exempts low bequests, or it first subsidizes them and then tax them, in which case the largest tax amount on low bequests cannot be larger than the supremum of the bequests that are subsidized.

An intermediary result in our analysis of the optimal tax function is that bequests should, on average, be taxed rather than subsidized. This coincides with a result of Piketty and Saez (2013), except that they reach this conclusion only for some distribution of normative weights. This further illustrates how important it is to be able to endogenize these weights as a function of fairness principles.

Three policy recommendations are consistent with our results. First, taxing bequests should be viewed as a way to redistribute from individuals who

[1] Piketty and Saez (2013) take advantage of the flexibility allowed by the theory of general social marginal welfare weights (Saez and Stantcheva, 2016) in order to evaluate different social welfare choice criteria, all of which are different from the one we derive.

inherited from their parents to those who were less lucky. That means that the tax/transfer scheme of bequests should bring some strictly positive surplus to the government, money that should be allocated to all individuals independently of how much they receive from their parents and how much they leave to their children.

Second, the trade-off between increasing the transfers to all and decreasing inheritance taxes should be solved by looking at how they affect the well-being of those who did not receive inheritance from their parents. Indeed, the most altruistic among them will prefer a decrease in inheritance tax (or an increase in inheritance subsidy) whereas the self-centered among them will prefer an increase in transfer.

Third, in spite of a positive average tax on bequests, there can be two reasons to subsidize low bequests. The first reason, reminiscent of results from the literature that we review below, is efficiency, because subsidizing may be a way to obtain a dominating distribution of bequests. The second reason is fairness, because it is a way to redistribute among the poorest individuals (the ones who did not get any bequests from their parents) from self-centered to altruistic individuals. This, however, goes with a precise condition: fairness can only justify subsidies in an interval that goes up to the bequests left by the poorest and most altruistic individuals.

The literature on inheritance taxation has raised a number of different issues, among which the issue of the shape of the tax function maximizing an inequality-sensitive social welfare function is a central one. This literature has not reached any consensus. This comes from differences in, first, the modeling of the interactions between parents and children, and, second, the objective that the egalitarian planner is supposed to follow. In many papers, individuals are assumed to have the same preferences but different abilities to earn income, with the consequence that the planner tries to redistribute from high to low-wage individuals. Because inheritance inequality does not reveal new information about individuals' wages, Farhi and Werning (2010) and Kaplow (2001), following a Atkinson and Stiglitz (1976) type of argument, proves that taxing labor income is more efficient than taxing inheritance. In a similar model, Kopczuk (2013a) makes the point that a countervailing force pushing in favor of bequests taxation is that receiving a large inheritance disincentivizes labor supply. Kaplow (1995), Farhi and Werning (2010) and Kopczuk (2013a) all make the point that subsidizing bequests is a way to incentivize parents to internalize the positive externality of giving.

In Piketty and Saez (2013), on the contrary, differences in bequests don't necessarily come from differences in parents' ability to earn income. They can come from parents' altruism, which differ across parents. As a result, taxing inheritance can be the most efficient way to redistribute from lucky to unlucky children. Moreover, Piketty and Saez (2013) do not divide individuals into parents and children. They rather consider the entire life time, so that all individuals are children and parents in turn. Fiscal policies then affect both the resources that individuals receive early in life and the tax they pay at the end of their life. The positive externality is now reflected in the level of sustainable tax and transfer policies. As a consequence, Piketty and Saez (2013) show that taxing bequests may end up being optimal.

As explained above, we keep the same intergenerational setting as Piketty and Saez (2013), but we characterize a specific social welfare function and we study tax functions in a larger domain. We prove that tax functions that first subsidize low bequests and then tax larger bequests can be optimal. These functions are not studied by Piketty and Saez (2013), but Kopczuk (2013a) conjectures that they might be optimal.

As discussed in reviews by Cremer and Pestieau (2006) and Kopczuk (2013b), the efficiency and fairness implications of inheritance taxation may depend on the bequest motive. For instance, accidental bequests, which exist in the absence of a perfect annuity market when parents die before consuming all their savings, can be taxed without any efficiency costs. We study inheritance taxation under a joy-of-giving motive, which explicitly acknowledges the desire that parents may hold to leave a bequest. The legitimacy of such desire is central in discussions surrounding the taxation of bequests (McCaffery, 1994). An alternative bequest motive consistent with this desire is altruism, whereby parents care for the *utility* of their child (whereas parents care only about the net-of-tax inheritance received by their child under a joy-of-giving motive). The altruistic motive is at the center of the Barro-Becker dynastic model, which has been widely studied in the literature on optimal capital/inheritance taxation. Most centrally, Chamley (1986) and Judd (1985) conclude that the tax on inheritances should be zero in the long run. More recently, Straub and Werning (2020) overturn this early results by showing that it only holds for high values of intertemporal elasticity of substitution, but otherwise such tax is positive and significant. We do not consider altruism and it remains an open question whether our results also hold in this alternative setting. One result that is very likely to carry through is that the inheritance tax should globally collect a non-negative amount. The reason is that any tax violating this would be dominated by Laissez-Faire, because such tax would hurt self-centered individuals whose parents are self-centered. We also note that the altruistic model of bequests has been empirically tested and rejected by Wilhelm (1996).

Some authors study the effect on the optimal tax of the fact that the number of children may differ across families and parents may decide not to give equal bequests to all their children, like Cremer et al. (2001). We could take that into account at least to some extent: the worst-off would remain the same under more general assumptions on the number of children.

Some other aspects of inheritance taxation are completely ignored in our analysis. Cremer et al. (2003) study capital income taxation as subsidiary to inheritance taxation in a world in which bequests may not be observable. Nordblom and Ohlsson (2006) study the possibilities to escape bequest taxation through inter-vivos gifts. Stantcheva (2015) studies inheritance taxation in its relationship to investments in human capital. Mirrlees et al. (2010) discuss the administrative cost implied by the collection of an inheritance tax. Golosov et al. (2003) and Kocherlakota (2005) study tax instruments that are allowed to vary over time, leaving more room for improving welfare. Fleurbaey et al. (2018) study the implications of ex-post egalitarianism for the taxation of accidental bequests resulting from premature mortality.

In Section 2, we describe the model, by insisting on the similarities and differences with the pioneer model of Piketty and Saez (2013). In Section 3, we discuss our social welfare function and the axioms that justify it. In Section 4, we study the optimal tax function and we state our main results. In Section 5, we provide some concluding comments. In Section 6, we develop the proofs of the results.

## 2 The model

We consider an economy with a discrete set of successive generations, $0, 1, \ldots$. Each generation contains a set $[0, 1]$ of individuals of measure 1. We use $\lambda$ to denote the probability measure on $[0, 1]$, that is the mass of individuals whose names are between $i$ and $j$ ($i, j \in [0, 1]$, $i < j$) is equal to $j - i$. We let $M[0, 1]$ denote the set of Lebesgue-measurable subsets of $[0, 1]$ and $\mu(J)$ denote the measure of $J \in M[0, 1]$.

Remember that, contrary to Piketty and Saez (2013), we are not interested into the trade-off between labor income taxation and bequest taxation. We only raise the subquestions of whether total lifetime labor income should be taxed (resp., subsidized) so as to subsidize (resp., tax) (at least some) bequest leavers. As a result, we assume that all individuals earn an identical lifetime income of $w$. Therefore, differences in lifetime budgets only come from the bequests individuals get (or not) at the beginning of their life. The assumption of equal $w$ among individuals, however, is far from necessary for our results. We come back on this issue in the conclusion.

Preferences are defined over lifetime consumption, $c$, and the inheritance received by their heir, $h$. Preferences are heterogeneous in the population. We make three assumptions on the *joy-of-giving* utility functions representing these preferences.

1. We assume preferences to be normal on both goods, consumption and inheritance, at all prices.
2. We assume that at each period there exist some selfish individuals, that is their utility function is
$$u^s(c, h) = c.$$
A consequence of this assumption is that at each period there exist some individuals who do not receive any inheritance.
3. Finally, we assume that at each period there exist individuals exhibiting the largest level of altruism. That is, some individuals have utility function $u^a$ and for all compact opportunity set $B \subset \mathbb{R}^2_+$, if bundle $(c^a, h^a)$ is the best bundle in $B$ according to $u^a$ and $(c_i, h_i)$ the best bundle according to any utility function $u_i$ of another individual then $h^a \geq h_i$. Observe that this assumption would be a consequence of imposing the classical single-crossing property (which amounts to assume that individuals can be ranked according to their level of altruism) and a kind of compactness of the domain of preferences. We don't need these assumptions and limit ourselves at imposing the existence of most altruistic individuals.

Individuals live one period, after which they are replaced by the individual of the same dynasty and the next generation. In addition to goods $c$ and $h$, it is convenient to consider the quantity of money that an individual receives at the beginning of her life, $g$, and the bequest left by an individual, $b$, which is the quantity of money that she does not consume at the benefit of her heir. Quantities $g$, $c$, $b$ and $h$ will be related to each other when we model taxation below, but we don't need to introduce these relations in the first step of our analysis, when we discuss the social welfare function.

Indeed, we begin by focusing on what happens to one generation. Anticipating that we will later restrict our attention to long-run equilibrium allocations, we restrict our attention to allocations in which the distribution of money received by this generation from the previous one, $g$, is identical to the distribution of money received by the following one, $h$. Formally, individual $i \in [0,1]$ consumes a bundle

$$z_i := (g_i, c_i, h_i) \in X = \mathbb{R}^3_+.$$

An *allocation* $z \in Z := X^{[0,1]}$ is a function $z : [0,1] \to X$. An allocation $z = (g_i, c_i, h_i) \in Z$ is a *steady-state* allocation if the distribution of the $g_i$'s is equal to the distribution of $h_i$'s, that is, if $(\hat{g}_i)_{i\in[0,1]} = (\hat{h}_i)_{i\in[0,1]}$, where $(\hat{x}_i)_{i\in[0,1]}$ denotes the permutation of $(x_i)_{i\in[0,1]}$ in which elements are ranked in increasing order. We let $S$ denote the set of steady-state allocations. A (one generation) *economy* is a profile of utility functions $u = (u_i)_{i\in[0,1]} \in \mathcal{U} = U^{[0,1]}$, where $U$ is the set of acceptable utility functions. In particular, $u^s, u^a \in U$.

# 3 Social welfare

In this section, we define the social welfare function (SWF) that we use in the next section to study the optimal tax and we discuss its axiomatic foundation.

The SWF works by applying the lexicographic aggregator to individual well-being indices that capture the fairness principles of the planner. Each index represents the preferences of an agent. Let us define this index first. It is illustrated in Fig. 1. Agent $i$ is consuming $(c_i, h_i)$. The indifference curve through $(c_i, h_i)$ shows that agent $i$ is indifferent between $(c_i, h_i)$ and maximizing her utility over a budget of slope $-R$ starting at $(c, 0)$, where $R$ is the exogenous rate of return on savings per generation. Budgets of slope $-R$ are first-best, or laisser-faire, budgets, that is, in the absence of taxation. To say it differently, this agent is indifferent between her actual consumption, $(c_i, h_i)$, and being free to allocate a wealth of $c$ between own consumption today and children's inheritance tomorrow in the absence of taxation. We state that this agent has a current well-being of $c$. The objective of the planner is to maximize the lowest well-being, and in case of a tie, to maximize the second lowest well-being, etc.

To define this SWF formally, we need the following notation. A *social ordering* is a complete ordering on steady-state allocations. A *Social Welfare Function (SWF)* is a function $\boldsymbol{R}$ associating each economy $u \in \mathcal{U}$ with a social ordering $\boldsymbol{R}(u)$.

We define the intertemporal budget set of agent $i$ with bundle $z_i = (g_i, c_i, h_i)$ when laissez-faire prevails as[2]

$$B^{LF}(z_i) := \left\{ z'_i = (g'_i, c'_i, h'_i) \in X \,\middle|\, c'_i + \frac{h'_i}{R} \leq c_i + \frac{h_i}{R} \right\}.$$

The well-being index we are interested in, which we denote as $u^c$ and we call $c$-equivalent utility, can be defined as follows.

[2]As mentioned above, $g_i$, $c_i$ and $h_i$ will be related to each other when we model taxation. Thus, even if $g_i$ does not appear in the inequality defining $B^{LF}$, $c_i$ and $h_i$ will depend on $g_i$ when we model taxation.

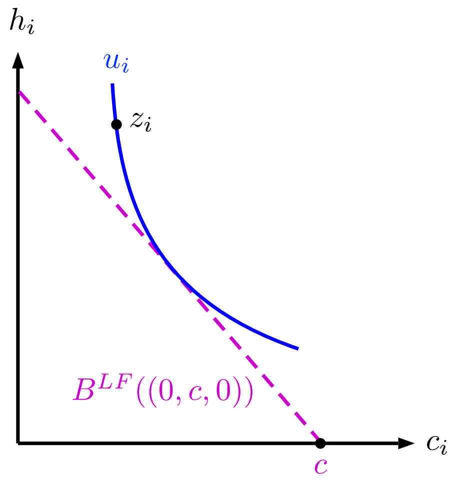


Figure 1: The $c$-equivalent utility

**Definition 1** ($c$-equivalent utility)**.**
*For all* $i \in [0,1]$, $z_i = (g_i, c_i, h_i) \in X$ *and* $u_i \in U$,

$$u^c(z_i, u_i) = c \Leftrightarrow u_i(c_i, h_i) = u_i \left( \arg\max_{u_i} B^{LF}\left((g_i, c, 0)\right) \right).$$

The SWF $\boldsymbol{R}^{c\text{–}lex}$ compares two allocations by applying the leximin aggregator to lists of $c$-equivalent utilities associated to the allocations.

**SOF 1** ($\boldsymbol{R}^{c\text{–}lex}$)**.** *For all* $u \in \mathcal{U}$ *and any two allocations* $z = (g_i, c_i, h_i)_{i\in[0,1]}, z' = (g_i', c_i', h_i')_{i\in[0,1]} \in S$

$$z' \ \boldsymbol{R}^{c-lex}(u) \ z \quad \Leftrightarrow \quad (u^c(z_i', u_i))_{i\in[0,1]} \geq_{lex} (u^c(z_i, u_i))_{i\in[0,1]}.$$

This way of measuring well-being has two key properties. First, it does not depend on $g_i$, that is the quantity of money one agent received as inheritance does not matter per se. The only thing that matters is the quantity of money that this agent and her child consume. Second, how precisely an agent allocates their wealth between own consumption and bequest does not matter, provided this agent allocates it freely. As a result, more altruistic or more selfish agents have the same $c$-equivalent utility when they allocate the same quantity of money in the absence of taxation.

The combination of these two properties is the basis on which the axiomatization of this SWF is grounded. Indeed, it satisfies the following three important axioms. The first one is the classical Pareto axiom. It requires weak social preference when all individuals weakly prefer one allocation over another. In addition, it requires strict social preference when one set (of positive measure) of individuals strictly prefer the former allocation.

**Axiom 1** (Pareto)**.**
*For all economy* $u \in \mathcal{U}$ *and steady-state allocations* $z = (g_i, c_i, h_i)_{i\in[0,1]}, z' =$

$(g'_i, c'_i, h'_i)_{i\in[0,1]} \in S$, *if for all* $i \in [0,1]$

$$u_i(c'_i, h'_i) \geq u_i(c_i, h_i)$$

*then* $z'$ $\boldsymbol{R}(u)$ $z$, *and if, in addition, there exists a subset of individuals* $J \in M[0,1]$ *such that* $\mu(J) > 0$ *and for all* $j \in J$

$$u_j(c'_i, h'_i) > u_j(c_j, h_j)$$

*then* $z'$ $\boldsymbol{P}(u)$ $z$.

The fact that $\boldsymbol{R}^{c\text{–}lex}$ satisfies Pareto comes from $c$-equivalent utility being identical at all points of the indifference curve. Actually, $c$-equivalent utility is a recalibration of the utility function.

The second axiom, compensation for children's lack of luck, in short Compensation, encapsulates the idea that individuals should not be held responsible for the lack of altruism of their parents, that is they should be compensated for receiving low inheritance. Formally, it requires that if two individuals with identical preferences consume bundles that dominate one another (that is, one individual has both a larger consumption and a larger inheritance received by her child) then a transfer from the richer to the poorer of these individuals is a strict social improvement. We add the restriction that individuals have identical preferences in order to avoid a classical impossibility with Pareto.

**Axiom 2** (Compensation)**.**
*For all economy* $u \in \mathcal{U}$, *steady-state allocations* $z = (g_i, c_i, h_i)_{i\in[0,1]}$, $z' = (g'_i, c'_i, h'_i)_{i\in[0,1]} \in S$, *subsets of individuals* $J, K \in M[0,1]$ *such that* $\mu(J) = \mu(K) > 0$, *and* $\delta \in (0, \frac{1}{2}]$, *if for all* $j, q \in J$ *and* $k, \ell \in K$,

- $u_j = u_q = u_k = u_\ell$, $c_j = c_q, c_k = c_\ell$, $h_j = h_q, h_k = h_\ell$,
- $c_j + \delta(c_k - c_j) = c'_j = c'_q \leq c'_\ell = c'_k = c_k - \delta(c_k - c_j)$,
- $h_j + \delta(h_k - h_j) = h'_j = h'_q \leq h'_\ell = h'_k = h_k - \delta(h_k - h_j)$,

*and* $z_i = z'_i$ *for all* $i \notin J \cup K$ *then* $z'$ $\boldsymbol{P}(u)$ $z$.

The fact that $\boldsymbol{R}^{c\text{–}lex}$ satisfies Compensation comes from $c$-equivalent utility being independent of the inheritance received $g$. As a consequence, equalizing consumption and bequest among agents with the same preferences is a way to make individual well-being independent of how much one inherited from their parents.

The third axiom, responsibility for parents' choices, in short Responsibility, encapsulates the idea that individuals should be considered responsible for their preferences, that is they should be free to allocate their wealth the way they wish. It requires that two individuals with the same inheritance from their parents should ideally be free to choose their preferred bundle in the same budget set of slope $-R$, the laisser-faire slope. Rather than requiring that they should choose in the same budget, the axiom requires that budget inequality between two such agents should be reduced.

**Axiom 3** (Responsibility).
*For all economy* $u \in \mathcal{U}$*, steady-state allocations* $z = (g_i, c_i, h_i)_{i\in[0,1]}$*,* $z' = (g'_i, c'_i, h'_i)_{i\in[0,1]} \in S$*, subsets of individuals* $J, K \in M[0,1]$ *such that* $\mu(J) = \mu(K) > 0$*, if there exists* $\delta > 0$ *such that for all* $j, q \in J$ *and* $k, \ell \in K$*,*

- $z_i \in \max_{|u_i} B^{LF}(z_i), \ \forall\, i \in \{j, q, k, \ell\}$,
  $z'_i \in \max_{|u_i} B^{LF}(z'_i), \ \forall\, i \in \{j, q, k, \ell\}$
- $y_j + \delta = y_q + \delta = y'_j = y'_q < y'_k = y'_\ell = y_k - \delta = y_\ell - \delta$,

*where*

$$y_i = c_i + \frac{h_i}{R}, y'_i = c'_i + \frac{h'_i}{R}, \forall\, i \in \{j, q, k, \ell\},$$

*and* $z_i = z'_i$ *for all* $i \notin J \cup K$ *then* $z'$ $\boldsymbol{P}(u)$ $z$*.*

The fact that $\boldsymbol{R}^{c\text{–}lex}$ satisfies Responsibility comes from $c$-equivalent utility assigning the same well-being to two agents as soon as they both freely allocate their wealth absent any taxation.

In Appendix 6.3, we formally prove that $\boldsymbol{R}^{c\text{–}lex}$ is the only SWF that satisfies these three axioms together with a consistency requirement of the SWF across economies. A number of important remarks have to be made at this stage.

1. Both Compensation and Responsibility are transfer axioms, that is, they are satisfied if a transfer of goods or of budget is implemented from the richer to the poorer agent, that is to say that they display a positive degree of inequality aversion. The SWF we come at, though, exhibits an infinite inequality aversion. The fact that the combination of Pareto, transfer and consistency axioms leads to a SWF exhibiting infinite inequality aversion is common in the literature and the underlying logics is now well understood. This is the main reason why we relegate the proof to the Appendix.

2. There is something less common, though, in the axiomatic foundation of $\boldsymbol{R}^{c\text{–}lex}$. Compensation and responsibility axioms, indeed, are typically incompatible with each other. It is therefore a surprise that they turn out to be compatible in this model. This comes from the fact that agents in this model are both parents and children (building on the novelty of Piketty and Saez (2013)) and each agent's utility is influenced both when she receives inheritance, so that her wealth increases, and when she is prevented from bequeathing/incentivized to bequeath money to her child. It can be illustrated through the following simple example. Assume an altruistic parent plans to bequeath some amount of money to her child, whereas a selfish parent with the same wealth does not wish to bequeath anything to her own child. As a result of the bequest, the child of the altruistic parent would turn out better-off than that of the selfish parent. On the other hand, if the altruistic parent is prevented from bequeathing, she will end up worse-off than the selfish parent. The solution to this paradox, assuming non-distortionary transfers can take place, would be to withdraw some wealth from both parents and to allocate it to the child of the selfish parent in compensation for the lack of inheritance, taking into account that the two children themselves can be required to give a part of their wealth in case other agents are worse-off. That is, the ideal non-distortionary allocation would be to equalize wealth of all agents of all

generations, independently of whether this wealth comes from bequest or redistribution. Useless to say, such an allocation is impossible to achieve through bequest taxation.

# 4 Optimal tax

In this section, we study the allocations that maximize our SWF among those that can be implemented by a bequest tax function and a demogrant. Contrary to what we did in the previous section, we now take account of the influence of the tax scheme on the transmission of bequests, as the behavior of members of one generation influences the well-being of members of the next generation. To take account of these long-run effects of the tax, we restrict our attention to tax functions and demogrants that do not depend on time and we look at the corresponding long-run allocations, that is the allocations obtained when the distribution of bequests is stabilized across generations.

More precisely, we assume that at time $t$, each individual $it$ (from dynasty $i$ living in generation $t$) receives inheritance $g_{it} \geq 0$ (which is a function of the bequest left from individual $it-1$) and demogrant $D$, so that their total resources are $w+D+g_{it}$. Individual $it$ chooses consumption $c_{it} \geq 0$ and bequest $b_{it+1} \geq 0$ under the budget constraint

$$c_{it} + b_{it+1} = w + D + g_{it}.$$

Bequests are taxed according to tax function $\tau$, so that amount $b_{it+1} - \tau(b_{it+1})$ is transferred to individual $it+1$, who receives

$$h_{it+1} = R(b_{it+1} - \tau(b_{it+1})),$$

where $R$ is the interest rate. We assume that $\tau(0) = 0$, because any other value would amount to transferring the same (negative or positive) amount to all, which is exactly what $D$ achieves.

The same process takes place at $t+1$, with inheritance $g_{it+1} = h_{it+1} \geq 0$. Starting conditions at time $t+1$ may, therefore, differ from those at time $t$. Note that, in this model, it is the money collected at time $t$ through $\tau$ that is used to fund demogrant $D$. This captures the fact that, had we consider a model in which individuals live for many periods, with a fraction of them born and dead at each period, what an agent gets out of the redistribution system at each period of her life, $D$, is funded by the taxes on bequests of the individuals that lived (and died) during this individual's own life. This modeling is particularly appropriate to our objective to study the trade-off between modifying the budgets of individuals through a demogrant, funded by taxes on bequests, or subsidies to bequests, at the price of a lower or even negative demogrant.

For a given *tax-demogrant scheme* $(\tau, D)$, an *equilibrium* allocation at time $t$ is an allocation $z_t = (z_{it})_{i\in[0,1]} = (g_{it}, c_{it}, h_{it+1})_{i\in[0,1]} \in Z$ for which all $i \in [0,1]$ choose in the budget set defined by this scheme and their inheritance $g_{it}$, i.e.

$$B^{\tau}(w+D+g_{it}, 0) := \left\{ (c_{it}, h_{it+1}) \in \mathbb{R}^2_+ \,\middle|\, h_{it+1} \leq R\left((w+D+g_{it} - c_{it} - \tau\left(w+D+g_{it} - c_{it}\right)\right) \right\},$$

implying for all $i \in [0,1]$ that

$$h_{it+1} = R\left(w + D + g_{it} - c_{it} - \tau\left(w + D + g_{it} - c_{it}\right)\right). \qquad (1)$$

A *long-run equilibrium* allocation for a tax scheme $(\tau, D)$ is a steady-state equilibrium allocation $z = (g_i, c_i, h_i)_{i\in[0,1]} \in S$ to which this sequence of equilibrium allocations at time $t$ may converge. That is, at a long-run equilibrium allocation, Eq. (1) holds and the profile of inheritances received, $(\hat{g}_i)_{i\in[0,1]}$, is equal to the profile of inheritances left, $(\hat{h}_i)_{i\in[0,1]}$.

We need some further assumptions to guarantee that long-run equilibrium allocations exist and that we are able to apply our SWF to them.

As Piketty and Saez (2013), we assume that the stochastic transmission of preferences across generations is such that the distribution of preferences remains constant through time. In the terms of the previous section, that means that economy $u \in \mathcal{U}$ is constant through time, up to some (measure preserving) permutation of $i \in [0, 1]$.

We also assume that the economy converges over time to a unique long-run equilibrium independent on the initial distribution of inheritances $(g_{i0})_{i\in[0,1]}$. Piketty and Saez (2012) show that this assumption is met in their framework under reasonable conditions. In particular, the average taste for bequest cannot be too strong and the stochastic transmission of preferences across generations must satisfy an ergodicity property.[3] Importantly, this property does NOT imply that all members of one dynasty have the same preferences. Some altruistic parents have selfish children and the converse is true as well.

Observe that $(\tau, D)$ may yield a long-run equilibrium allocation in which not enough money is collected through tax $\tau$ to fund demogrant $D$. We need to further restrict our attention to tax schemes that meet the government budget constraint

$$D \leq \int_i \tau\left(g_i + w + D - c_i\right) di. \tag{2}$$

A demogrant $D$ is *sustainable* for the tax $\tau$ if the long-run equilibrium allocation associated to $(\tau, D)$ satisfies Eq. (2).[4] When this is the case, we say that the tax-demogrant scheme $(\tau, D)$ is sustainable. Observe that a given tax may admit several sustainable demogrants. For instance, both a zero demogrant and a negative demogrant are sustainable under a linear tax with rate zero.

A sustainable tax-demogrant scheme $(\tau, D)$ is *optimal* if there is no other sustainable tax scheme whose associated long-run equilibrium allocation is preferred by SWF $\boldsymbol{R}^{c\text{–}lex}$ to that associated to $(\tau, D)$. A tax $\tau$ is *optimal* in some domain if there is no alternative tax $\tau'$ in that domain for which the long-run equilibrium allocation associated to a sustainable scheme $(\tau', D')$ is preferred by SWF $\boldsymbol{R}^{c\text{–}lex}$ to the long-run equilibrium allocation associated to all sustainable schemes $(\tau, D)$.

An individual is among the *worst-offs* if all other individuals (in the long-run equilibrium generation) have a $c$-equivalent utility at least as large as this individual.

*Laissez-Faire* is a tax-demogrant scheme defined by a zero tax and a zero demogrant: $\tau^{LF}(b) = 0\,\forall b \geq 0$ and $D^{LF} = 0$.

[3] See Piketty and Saez (2013), page 1854.

[4] This sustainability constraint allows us to link the government and individuals budget constraints in the following way. A sustainable $(\tau, D)$ is optimal only if $D \leq \int_i \tau\left(g_i + w + D - c_i\right) di$ and $h_i = R\left(w + D + g_i - c_i - \tau\left(w + D + g_i - c_i\right)\right)$. These two equations give us the following sustainability constraint: $D \leq \int_i g_i + w + D - c_i - \frac{h_i}{R} di \Leftrightarrow \int_i c_i + \frac{h_i}{R} di \leq \int_i g_i + wdi$.

## 4.1 A positive average tax on bequests

Our first result answers the following question: at the optimal tax, should individuals' incomes be taxed so that bequests can be, on average, subsidized, or should bequests be taxed, on average, so as to subsidize individuals' incomes (through a demogrant)? The answer is that bequests should be taxed: the amount globally collected by an optimal inheritance tax cannot be negative. If subsidies are provided for some bequest levels, they must be paid for by taxes collected at other bequest levels. This answer is the opposite to that of Atkinson and Stiglitz (1976), Kaplow (2001) and Farhi and Werning (2010), confirming the crucial importance of taking account of the influence of taxing bequests on the inheritance distribution, and, therefore, the wealth of the parents. The optimal formula of Piketty and Saez (2013), on the other hand, is consistent with taxing bequests on average, depending on the distribution of normative weights. Our result shows that the distribution of normative weights that follows from imposing the axioms we propose unambiguously leads to a positive average tax rate on bequests.

The proof goes by comparing the optimal tax scheme with Laissez-Faire. Under Laissez-Faire, any individual $i$ freely allocates her lifetime resources between consumption and bequest, implying that her $c$-equivalent utility is equal to her lifetime resources $(w + g_i)$. Thus, the worst-off individuals are those with $g_i = 0$, and they all have a well-being level equal to $w$. Now, under any tax-demogrant scheme $(\tau, D)$, the $c$-equivalent utility of any *self-centered* individual is also equal to her consumption $(w+D+g_i)$. Provided that at least one individual who inherits nothing is self-centered,[5] an assumption that we impose (see assumption A1 below), her $c$-equivalent utility is equal to $w + D$. If the demogrant is negative, then her well-being is smaller than the well-being of the worst-off under Laissez-Faire. The result follows from the fact that our SWF ranks tax-demogrant schemes by comparing the long-run equilibrium well-being of the worst-off. To sum up, redistribution cannot take place from the general population towards those who leave some bequests, because such a redistribution hurts those who did not receive anything from their parents and do not plan to leave anything to their children either, and these ones are among the worst-offs.

The property of Laissez-Faire that all individuals for whom $g_i = 0$ are among the worst-offs and they all have the same $c$-equivalent utility is shared by all tax schemes $(\tau^*, D^*)$ in which $\tau^*$ exempts the bequests left by those who inherit nothing. This suggests that if $\tau^*$ maximizes the sustainable demogrant $D^*$ under the constraint that $\tau^*$ exempts the bequests left by those who inherit nothing, $(\tau^*, D^*)$ is a strong candidate to be optimal.

Indeed, our second result identifies a necessary condition on the optimal $(\tau', D')$ to be different from $(\tau^*, D^*)$: it needs to be the case that $D' \geq D^*$. It is an easy consequence of what we already said. If $(\tau', D')$ has $D' < D^*$, then the $c$-equivalent utility of a self-centered individual who did not inherit anything is lower at $(\tau', D')$, where it is equal to $w + D'$, than at $(\tau^*, D^*)$, where it is equal to $w + D^*$ and where this individual is among the worst-offs.

To define the largest bequest left by a zero-inheritor precisely, we impose the

[5] Piketty and Saez (2013) document that about half the population in France and the US receives negligible bequests in 2010, which suggests that some individuals in those countries do not enjoy leaving a bequest.

assumption that there are zero-inheritors with the most altruistic preferences. The needed assumption for the following proposition is, therefore:

Assumption **A1**: In any long-run equilibrium allocation, there are two disjoint subsets $I^s, I^a \subset [0,1]$ with $\mu(I^s) > 0$ and $\mu(I^a) > 0$ such that for all $s \in I^s$ and all $a \in I^a$ we have $g_a = g_s = 0$, $u_a = u^a$ and $u_s = u^s$.

Our first proposition summarizes the discussion above.

Proposition 1 makes use of the following definition. Let $b_a^{LF}(w+D)$ denote the optimal bequest left by individual $a \in I^a$ under a scheme $(\tau, D)$ that provides an exemption strictly larger than $b_a^{LF}(w+D)$, i.e.

$$b_a^{LF}(w+D) = \arg\max_{\tilde{b}_a \geq 0} u_a(w + D - \tilde{b}_a, R\tilde{b}_a).$$

**Proposition 1.** *(i) Under A1, a tax-demogrant scheme $(\tau, D)$ is optimal only if $D \geq 0$. (ii) Under A1, a tax-demogrant scheme $(\tau^*, D^*)$ that provides an exemption up to $b_a^{LF}(w+D^*)$ is optimal if there is no other sustainable tax-demogrant scheme $(\tau', D')$ such that $D' \geq D^*$.*

*Proof.* The proof is relegated in Appendix 6.1. ∎

The condition $D' \geq D^*$ is necessary for $(\tau', D')$ to be optimal, it is of course not sufficient. In the long-run equilibrium allocation associated to $(\tau^*, D^*)$, all zero-inheritors are equally well-off, and the optimal tax scheme needs to make all of them at least as well-off as at $(\tau^*, D^*)$.

Note that the long-run equilibrium allocation associated with the optimal tax scheme depends on the preference profile considered, preferences are heterogeneous and the long-run equilibrium profile of inheritances received is *endogenous* to the shape of the tax. Even without being able to characterize this allocation exactly, we derive in the next section some constraints on the shape of an optimal tax.

## 4.2 Tax exemption on low bequests, or limited subsidies and taxes

In the previous section, we proved that $D$, the demogrant, cannot be too low, and, in particular, cannot be negative. The demogrant should be thought of as the average amount transferred from those who leave a bequest for their children to the general population. In this section, we prove that $D$ cannot be too *large*, either. The intuition for this result is the following one. Worst-off individuals are to be found among the zero-inheritors. Among them, the well-being of the self-centered individuals is entirely determined by $D$. It is not the case for the other zero inheritors. In particular, the most altruistic among them, individuals $a$, receive $D$ but they pay $\tau(b_a)$, in which $b_a$ stands for their bequest. Proposition 1 implies that $D \geq \tau(b_a)$: individuals $a$ cannot be strict contributors to the tax system. It suggests that even if $a$ end up with a lower well-being than self-centered individuals, the difference in well-being should be limited.

This has the following implication for the optimal tax scheme. Let $b_a$ denote the bequest of these individuals $a$ at $(\tau, D)$. Let $\beta$ be a positive bequest level smaller than $b_a$ and let $\Delta$ be an amount of money smaller than $\beta$. Consider the alternative tax scheme $(\tau', D')$ satisfying

$$\begin{aligned} \tau'(b) &= \tau(b+\Delta) - \Delta, \quad \forall b \geq \beta - \Delta \\ D' &= D - \Delta, \end{aligned}$$

which is illustrated in Figure 2. Facing $(\tau', D')$, individuals $a$ do not see any difference between $(\tau', D')$ and $(\tau, D)$: the decrease in their lifetime income is perfectly compensated by the decrease in the tax they pay on their bequest. Zero-inheritor-self-centered individuals, on the contrary, are affected by the change, as their well-being decreases by $\Delta$. If, moreover, $(\tau', D')$ leaves money on the table, which is quite likely because $D$ decreases, this money can be redistributed to the entire population, thereby making individuals $a$ better-off. That illustrates that decreasing $D$ and decreasing the tax below some threshold is a policy tool to increase the well-being of $a$ at the expense of $s$ (the zero-inheritor-self-centered individuals).

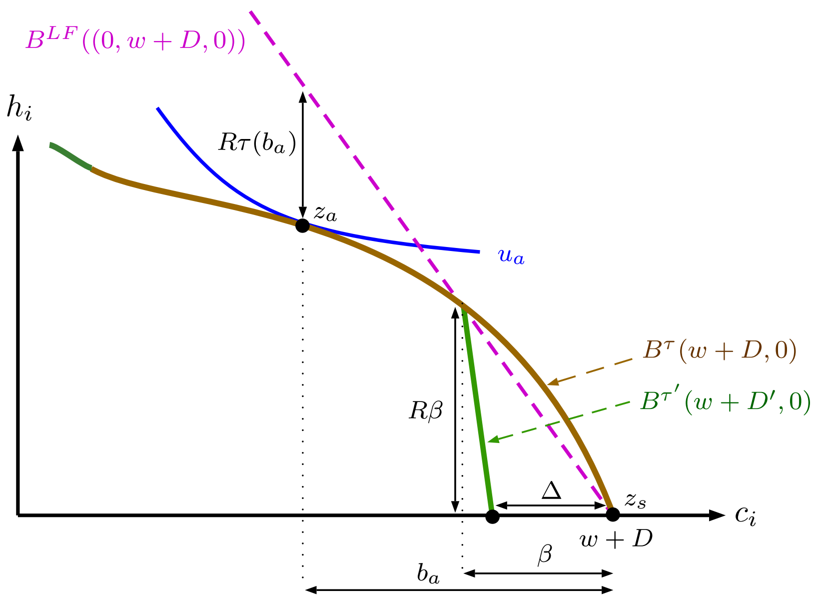


Figure 2: Tax-demogrant schemes $(\tau, D)$ and $(\tau', D')$ provide equal $c$-equivalent utility to the worst-off individuals $a$ even if the latter has a smaller demogrant.

This illustration is too simple, however, as $(\tau', D')$ is typically non-sustainable. To prove Proposition 2, we do identify a truncation of $(\tau, D)$ that is sustainable and that increases social welfare as soon as $D$, or, equivalently, $\tau(b_a)$, is too large.[6] As a consequence, we show in the next proposition that

1. tax functions with positive taxes on small bequest amounts are not optimal,

[6] When defining this truncation below, we give a precise value to $\beta$ and to $\Delta$.

2. monotonically increasing tax functions are not optimal unless they provide an exemption up to the amount of bequest that would be chosen under Laissez-Faire by individuals $a$ (who inherit nothing and have the most-altruistic preference),

3. a positive tax on the amount of bequest left by $a$ is not excluded though, at least when subsidies are provided on smaller bequest amounts, and

4. even when the optimal tax function subsidies smaller bequest amounts, the tax on the bequest amount left by $a$ must be limited (see Eq. (3) in the proposition below).

To proves these claims, we restrict our attention to tax functions $\tau$ for which $-\tau$ is *single-peaked.* This domain contains all tax functions that are policy relevant. In particular, this domain is the union of the two most relevant subdomains. The first subdomain contains all (weakly) monotonically increasing tax functions $\tau$ (bequests are taxed at an increasing non-negative rate), in which 0 is a peak for $-\tau$ (there may be an entire interval of peaks, in which case $\tau$ exempts bequests on this interval). The second subdomain contains all tax functions $\tau$ that are first monotonically (weakly) decreasing (small bequests are subsidized) and then monotonically (weakly) increasing. In this subdomain the peaks are all positive. We refer to the latter subdomain as that of *positive peak* tax functions, and to the former as that of (weakly) monotonically increasing tax functions. Observe that, by Proposition 1, any monotonically decreasing tax function is dominated because such tax cannot sustain a non-negative demogrant. We don't comment on these functions anymore.

In this domain, the worst-off individuals are either individuals $a$ or $s$. As illustrated in Figure 3, the reason is that the consumption of any zero-inheritor is at least as large as the consumption of $a$, but not larger than the consumption of $s$. As a result, their $c$-equivalent utility cannot be smaller than both the $c$-equivalent utilities of $s$ and $a$. Since either $s$ or $a$ are among the worst-offs, we can restrict the normative analysis to these two individuals. When a tax function $\tau$ implies a tax on the amount of bequest left by $a$, this individual is among the worst-offs whereas $s$ is not. Welfare would be improved if it were possible to increase the well-being of $a$ while keeping the well-being of $s$ above the well-being of $a$. The difficulty here is to make sure that such improvement materializes in the new long-run equilibrium allocation, which depends on the maximal demogrant that can be sustained.

Two additional mild assumptions are required for these results. First, when they consume at least as much as their labor income, the preferences of individuals who are *not* self-centered and consume at least $w$ must be strictly increasing in the inheritance received by their child. This assumption will guarantee that there exists a sufficiently large subsidy rate that induces these individuals to leave at least a threshold amount to their children.

Assumption **A2**: For all $i \in [0,1]$ with $u_i \neq u^s$ we have that $u_i(c_i, h_i)$ is strictly monotonic in $h_i$ when $c_i \geq w$.

Second, the long-run equilibrium amount collected by a tax-demogrant scheme is continuous in the demogrant.

Assumption **A3**: For all tax $\tau$, the amount of tax collected, i.e.

$$\int_i \tau(b_i)\, di,$$

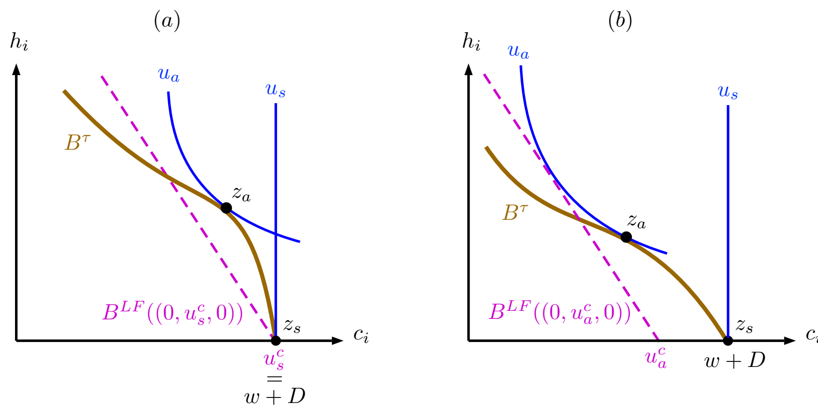


Figure 3: (a) Individuals $s$ with $g_s = 0$ and $u_s = u^s$ are among the worst-offs, where $u_s^c = u^c(z_s, u_s)$. (b) Individuals $a$ with $g_a = 0$ and $u_a = u^a$ are among the worst-offs, where $u_a^c = u^c(z_a, u_a)$.

where $b_i$ is the long-run equilibrium bequest left under $(\tau, D)$, is continuous in $D$.

Assumption A3 implies that, if $(\tau, D)$ is sustainable and leaves money on the table, then for some $D' > D$ the tax-demogrant scheme $(\tau, D')$ is also sustainable. Observe that assumption A3 is not necessary in the proposition below for the constraint derived on monotonically increasing tax functions.

We introduce the following notation. Let $\underline{b} \geq 0$ be the minimal bequest amount above which the tax $\tau$ provides zero subsidy, i.e.

$$\underline{b} = \min \tilde{x} \in \mathbb{R}_+ \text{ such that } \tau(x) \geq 0 \text{ for all } x \geq \tilde{x}.$$

Obviously, amount $\underline{b}$ is endogenous to the tax considered. For any positive peak tax, we have $\underline{b} > 0$ and we construct the alternative tax scheme $(\tau', D')$ using $\beta = \underline{b}$. For any monotonically increasing tax, we have $\underline{b} = 0$ and we construct the alternative tax scheme $(\tau', D')$ using some $\beta > \underline{b}$.

Proposition 2, our main result, provides us with the formal statements from which the four claims above are deduced.

**Proposition 2.** *Consider any tax $\tau$ for which $-\tau$ is single-peaked. Let $\underline{b}$ be the minimal bequest amount above which no subsidies are provided under $\tau$. Let $D^{max}$ be the maximal sustainable demogrant under $\tau$. Let $a$ be an individual who inherits nothing and holds the most altruistic preference. Let $b_a$ be the equilibrium bequest left by $a$ under $(\tau, D^{max})$. Let $b_a^{LF}(w)$ be the equilibrium bequest left by $a$ under Laissez-Faire.*

*(i) Under A1 and A2, if $\tau$ is monotonically increasing, then $\tau$ is optimal only if $\tau$ provides an exemption up to $b_a^{LF}(w)$.*

*(ii) Under A1, A2 and A3, $\tau$ is optimal only if*

$$\tau(b_a) \leq \underline{b}. \tag{3}$$

*Proof.* The proof is relegated in Appendix 6.2. ∎

An important feature of Proposition 2 is that the shape of monotonically increasing taxes is completely characterized up to the minimal amount exempted. Importantly, this amount is *exogenous* to the optimal tax. This amount only depends on the preference $u^a$, the interest rate $R$ and the wage rate $w$. This implies this shape is valid regardless of the exact preferences profile defining the economy. This contrasts with characteristics of optimal tax as derived in the literature (Piketty and Saez, 2013), which typically depend on statistics endogenous to the optimal tax. On the contrary, the bequest amount $b_a$ in claim (ii) of Proposition 2 is also endogenous to the tax.

To conclude this section, we note that our two propositions have an interesting corollary, namely that linear taxes are never optimal.

**Corollary 1.** *Under A1 et A2, no linear tax different from Laissez-Faire is optimal.*

*Proof.* Linear tax with negative rates cannot sustain non-negative demogrants. By Proposition 1, any tax scheme based on a negative demogrant is not optimal. Therefore, linear tax with negative rates are not optimal. Linear tax with positive rates are monotonically increasing and do not exempt bequests up to $b_a^{LF}(w)$. By Proposition 2, these tax functions are not optimal. ∎

# 5 Conclusion

The model that we study in this paper is designed to focus on the trade-off between subsidizing bequests or transferring a demogrant to all. A number of simplifying assumptions have been needed, on which we now come back.

We assumed away the issue of taxing labor incomes. We assume that all individuals have the same labor time and the same lifetime income, so that fairness does not require to redistribute labor income. Our result that bequests should, on average, be taxed so as to transfer a demogrant to all does not, therefore, come from the need to alleviate income inequalities, but only from the need to compensate children of selfish parents while preserving the parents' freedom to allocate their lifetime income the way they wish. As a consequence, our conclusions are compatible with heterogeneity of wages and labor times and the existence of a labor income tax system maximizing social welfare. This claim, however, calls for two qualifications.

First, our formal analysis can be replicated in a more general model only under the assumption that individuals' lifetime incomes are not influenced by the design of our bequest taxation system. In case this assumption is not valid, we should study the income redistribution and bequest taxation systems simultaneously. This task does not look feasible. Intuitively, though, the result of such an exercise is likely to remain that bequests are, on average, taxed, so as to compensate the children of self-centered parents while maintaining a sufficiently high utility level to self-centered individuals who did not receive anything from their own parents. Identifying who are the worst-off individuals and dealing with sustainability issues, however, would become much harder.

The second qualification has to do with the identification of the worst-off individuals in the case in which lifetime incomes are not influenced by the bequest taxation system. Given that the labor income taxation system aims at redistributing from higher wage individuals to lower wage individuals, it is extremely likely that the worst-off have to be found among the minimal-wage individuals. Consequently, our assumption A1 has to be strengthened into the existence, in any long-run equilibrium allocation, of individuals who did not inherit anything from their parents, who have the minimum wage and who have either the most altruistic or self-centered preferences. As a result, Proposition 2, part (i), for instance, would become that bequests should be exempted from taxes up to the amount left by the most altruistic individuals who did not receive any bequest from their parents and worked all their life at the minimum wage.

Our main results do not give us a formula that can be calibrated, but yet they can be used to qualitatively assess current tax systems. According to a recent report (see OCDE (2021)) 12 out of the 36 OECD countries do not tax bequests. Our Proposition 1, part (i) implies that this can not be optimal given our social welfare function. All the 24 countries that do tax bequests to children have a system consistent with the optimal tax system of Proposition 1, Part (ii) and Proposition 2, part (i): exemption for small bequests and a positive tax on larger ones. The interval of exemptions considerably vary across countries, from \$17,133 in Belgium to \$11,580,000 in the US (numbers in 2020 USD). While the former amount is likely to be smaller than the bequest left by the most altruistic parents having not received anything from their own parents and having worked at the minimum wage, the latter amount is clearly above this threshold. The money collected through bequest taxes is below 2% of the total fiscal revenues in all countries, and even below 1% in most countries, suggesting that the corresponding demogrant is not maximized. More research is needed, however, to compute the optimal tax systems in these countries.

In the model, we also assumed that the number of children is identical across households. Allowing heterogeneity among the number of children would not change the fact that the worst-off individuals have to be found among those who did not inherit anything from their parents. A new question would emerge, however, regarding the amount of exempted bequest. It would still be defined with reference to the amount left by the most altruistic individuals who did not receive any bequest from their parents, but the choice is between considering the bequests of parents of the largest number of children or with only one child. The former choice is appropriate if parents are modeled as caring about the per capita bequest received by their children. The latter choice is appropriate if parents are modeled as caring about the total bequest left.

The interest rate, $R$, is exogenous in our model. This is typical of a small open economy. If it is endogenous, but further assumptions make it depend only on the distribution of preferences in the economy, then our analysis carries over with $R$ being replaced with the endogenous rate. If, on the contrary, the interest rate may vary across time, then our results change. The intuition is that our optimal tax scheme needs to be amended so as to redistribute further from the lucky ones who face higher interest rates towards those who face lower interested rates. Moreover the leximin nature of our SWF implies that the worst-offs belong to those who did not inherit anything from their parents. Therefore, whether an individual is lucky or not only depends on the future interest rates. So, an optimal tax system should redistribute from those who can save for their children at a high rate towards those who save at a low rate. How precisely this should be done requires additional research.

Other assumptions would be much more difficult to relax. They would require to redefine the social welfare function or the policy tools. It would be the case, for instance, if individuals are interested in the entire lifetime of their children and not only how much they bequeath, in which case an increase of the demogrant benefits the altruistic parents more than the self-centered ones, if they have unequal life expectancy, in which case the social planner may wish to subsidize the bequest of short-lived individuals, if fertility choices are constrained, in which case the social planner may wish to favor those who wanted to have children but could not, and, therefore, do not leave any bequest, if children can inherit from different adults, raising the question of whether the tax should be donor-based or recipient based, if bequests are only partially observable, etc.

# 6 Appendix

## 6.1 Proof of Proposition 1

First, we prove claim (i). We show that any $(\tau, D)$ with $D < 0$ is dominated by Laissez-Faire.

We start by showing for the long-run equilibrium allocation $z^{LF} \in S$ associated to Laissez-Faire that $u^c(z_i^{LF}, u_i)$ for all $i \in [0, 1]$. Under Laissez-Faire, any $i \in [0, 1]$ choses in the budget set

$$B^{LF}((0, w + g_i^{LF}, 0)),$$

implying that $u^c(z_i^{LF}, u_i) = w + g_i^{LF}$.

We then show for the long-run equilibrium allocation $z \in S$ associated to $(\tau, D)$ that some subset $J \subset [0, 1]$ with $\mu(J) > 0$ is such that $u^c(z_j, u_j) = w + D$ for all $j \in J$. By assumption A1, there is a subset $J \subset [0, 1]$ with $\mu(J) > 0$ such that $g_j = 0$ and $u_j = u^s$ for all $j \in J$. Since these individuals are self-centered, we have that $z_j = (0, w + D, 0)$ and $u^c(z_j, u_j) = w + D$.

As $D < 0$, this shows that $u^c(z_j, u_j) < w$ for all $j \in J$, showing that Laissez-Faire is preferred to $(\tau, D)$ by $\boldsymbol{R}^{c\text{–}lex}$.

Second, we prove claim (ii). We show that any $(\tau'', D'')$ with $D'' < D^*$ is dominated by $(\tau^*, D^*)$.

We start by showing for the long-run equilibrium allocation $z^* \in S$ associated to $(\tau^*, D^*)$ that $u^c(z_i^*, u_i) \geq w + D^*$ for all $i \in [0, 1]$. In equilibrium, any $i \in [0, 1]$ choses in the budget set $B^{\tau^*}(w + D^* + g_i^*, 0)$. For all $i, j \in [0, 1]$ with $u_i = u_j$ and $g_j^* = 0$ we must have $u_i(z_i^*) \geq u_j(z_j^*)$ because $B^{\tau^*}(w + D^*, 0) \subseteq B^{\tau^*}(w + D^* + g_i^*, 0)$. As $u_i = u_j$, this implies that $u^c(z_i^*, u_i) \geq u^c(z_j^*, u_j)$. There remains to show that $u^c(z_j^*, u_j) = w + D^*$. As $\tau^*$ provides an exemption up to $b_a^{LF}(w + D^*)$, any $j \in [0, 1]$ with $g_j^* = 0$ choses the same bundle $z_j^*$ in her budget set $B^{\tau^*}(w + D^*, 0)$, as she would chose in the Laissez-Faire budget set

$$B^{LF}((0, w + D^*, 0)),$$

implying that $u^c(z_j^*, u_j) = w + D^*$.

We then show for the long-run equilibrium allocation $z'' \in S$ associated to $(\tau'', D'')$ that some subset $J \subset [0, 1]$ with $\mu(J) > 0$ is such that $u^c(z_j'', u_j) = w + D''$ for all $j \in J$. By assumption A1, there is a subset $J \subset [0, 1]$ with $\mu(J) > 0$ such that $g_j'' = 0$ and $u_j = u^s$ for all $j \in J$. Since these individuals are self-centered, we have under the long-run equilibrium allocation $z'' \in S$ associated to $(\tau'', D'')$ that $z''_j = (0, w + D'', 0)$ and so $u^c(z''_j, u_j) = w + D''$.

As $D'' < D^*$, this shows that $u^c(z''_j, u_j) < w + D^*$ for all $j \in J$, showing that $(\tau^*, D^*)$ is preferred to $(\tau'', D'')$ by $\boldsymbol{R}^{c\text{–}lex}$.

## 6.2 Proof of Proposition 2

Proposition 2 provides a limit on the tax paid on the amount of bequest left by $a$. The proof constructs another sustainable tax-demogrant scheme that dominates $(\tau, D)$. The construction is based on a sustainable tax-demogrant scheme $(\tau^\Delta, D - \Delta)$ illustrated in Figure 4. As shown in the figure, this second scheme linearly truncates the budget set faced by individuals under $(\tau, D)$. This truncation reduces the demogrant by an amount $\Delta$, but the new tax $\tau^\Delta$ provides large subsidies on small bequests. Provided the inheritance they receive is unchanged, individuals chose the same bundle under both schemes, at least if this bundle does not lie in the truncated part of their budget set. This implies that the well-being of $a$ is the same under both schemes. The smaller demogrant reduces the well-being of $s$, but $s$ still has a larger well-being than $a$. In the proof, we show that when $(\tau, D)$ is sustainable, scheme $(\tau^\Delta, D - \Delta)$ leaves money on the table. Money is left on the table because (i) the new scheme saves on self-centered individuals who receive a smaller demogrant and (ii) the new scheme does not spend more on altruistic individuals because its subsidies on small bequests are financed by the reduction in the demogrant. Then, the money saved by the new scheme can finance a small increase in the demogrant, such that $(\tau^\Delta, D - \Delta + \epsilon)$ is sustainable for some $\epsilon > 0$. The larger demogrant increases the well-being of all individuals, including that of the worst-off.

Here is the intuition why scheme $(\tau^\Delta, D - \Delta)$ is sustainable. The truncation creates a kink in the budet set faced by individuals under $(\tau, D)$. Importantly, this kink is located at an amount of bequest $\underline{b}$ above which the tax $\tau$ is non-negative and monotonically increasing. The larger the rate of subsidies on small bequests associated to $\tau^\Delta$, the more numerous the altruistic individuals who "bunch" at the kink, at least for those who used to leave a bequest smaller than $\underline{b}$ under $(\tau, D)$. If all of these individuals "bunch" at the kink, then the long-run equilibrium profile of inheritances under $(\tau, D)$ would be first-order stochastically dominated by the long-run equilibrium profile of inheritances under $(\tau^\Delta, D - \Delta)$. We can then show that $(\tau^\Delta, D - \Delta)$ is sustainable if $(\tau, D)$ is sustainable because, above the kink, the tax paid is increasing in the bequest left.

Both claims (i) and (ii) in Proposition 2 rely on the following two lemmas. Recall that we denote by $a$ an individual for whom the long-run equilibrium $g_a = 0$ and for whom $u_a = u^a$, and by $s$ an individual for whom the long-run equilibrium $g_s = 0$ and for whom $u_s = u^s$.

**Lemma 1.** *Under A1, for any tax-demogrant scheme $(\tau, D)$ for which $-\tau$ is single-peaked, either $a$ or $s$ are among the worst-offs.*

*Proof.* Let $z \in S$ denote the long-run equilibrium allocation associated to $(\tau, D)$. By assumption A1, there exist two individuals $a$ and $s$ with $g_a = 0$, $u_a = u^a$, $g_s = 0$ and $u_s = u^s$. We derive a contradiction when assuming that for some $k \in [0, 1]$ we have $u^c(z_k, u_k) < u^c(z_a, u_a)$ and $u^c(z_k, u_k) < u^c(z_s, u_s)$.

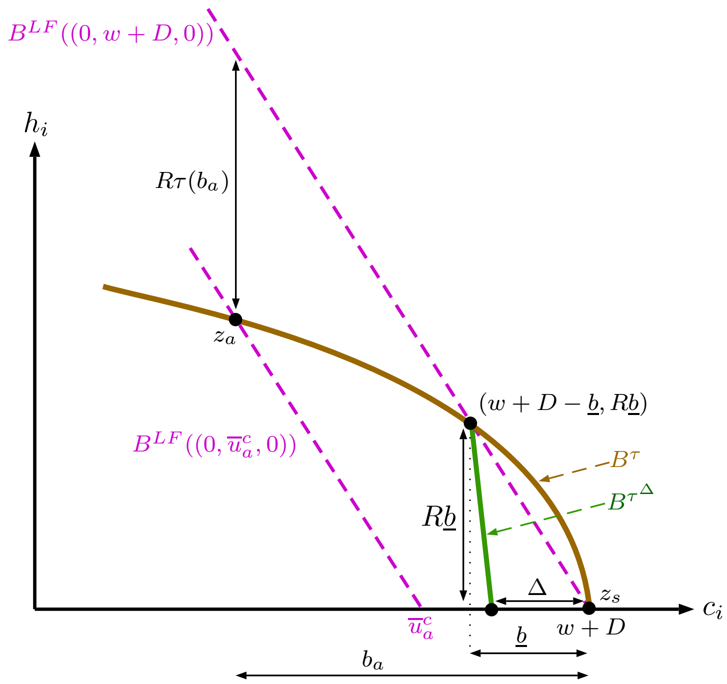


Figure 4: The tax-demogrant scheme $(\tau, D)$ is dominated because the tax function $\tau$ taxes too much the bequest $b_a$ left by $a$. Individual $a$ is the worst-off because $u^c(z_a, u_a) \leq \overline{u}_a^c$. The sustainable scheme $(\tau^\Delta, D - \Delta)$ has a smaller demogrant, does not affect $u^c(z_a, u_a)$ and leaves money on the table.

Under $z$, any $i \in [0, 1]$ choses in her budget set $B^\tau(w + D + g_i, 0)$. This implies for all $j \in [0, 1]$ with $u_j = u_k$ and $g_j = 0$ that $u_j(z_j) \leq u_k(z_k)$, and hence $u^c(z_j, u_j) \leq u^c(z_k, u_k)$. Thus, we can assume without loss of generality that $g_k = 0$.

By definition of $a$ and $s$, the fact that $g_a = g_k = g_s = 0$ implies that $c_a \leq c_k \leq c_s = w + D$. Letting $u_k^c = u^c(z_k, u_k)$, we show that $z_k \notin B^{LF}((0, u_k^c, 0))$. There are two cases, which are illustrated in Figure 5.

- Case 1: $z_s \in B^{LF}((0, u_k^c, 0))$.

  Let $z_s' = \arg\max_{u_s} B^{LF}((0, u_k^c, 0))$. This case is such that $u_s(z_s) \leq u_s(z_s')$. As by definition $u^c(z_s', u_s) = u_k^c$, we have that $u^c(z_s, u_s) \leq u_k^c$ and thus $u^c(z_s, u_s) \leq u^c(z_k, u_k)$, a contradiction.

- Case 2: $z_s \notin B^{LF}((0, u_k^c, 0))$.

  As $z_s = (0, w + D, 0) \notin B^{LF}((0, u_k^c, 0))$ and $g_k = g_s = 0$, we have $z_k \notin B^{LF}((0, u_k^c, 0))$ unless $\tau(w + D - c_k) \geq 0$.[7] As $-\tau$ is single-peaked and

[7]By definition, $B^{LF}((0, u_k^c, 0)) = \{(c_i, h_i) \mid c_i + h_i/R \leq u_k^c\}$. When $z_s = (0, w + D, 0) \notin B^{LF}((0, u_k^c, 0))$, we have $u_k^c < w+D$. Since in equilibrium, $h_k = R(w+D-c_k-\tau(w+D-c_k))$, we have $c_k + h_k/R < w + D$ only if $\tau(w + D - c_k) > 0$.

$c_a \leq c_k$, this implies in turn that $\tau(w + D - c_a) \geq \tau(w + D - c_k)$. Since $g_k = g_a = 0$, the fact that $\tau(w + D - c_a) \geq \tau(w + D - c_k) \geq 0$ and $z_k \in B^{LF}((0, u_k^c, 0))$ together imply that $z_a \in B^{LF}((0, u_k^c, 0))$. Then, $u^c(z_a, u_a) \leq u_k^c$ and thus $u^c(z_a, u_a) \leq u^c(z_k, u_k)$, a contradiction.

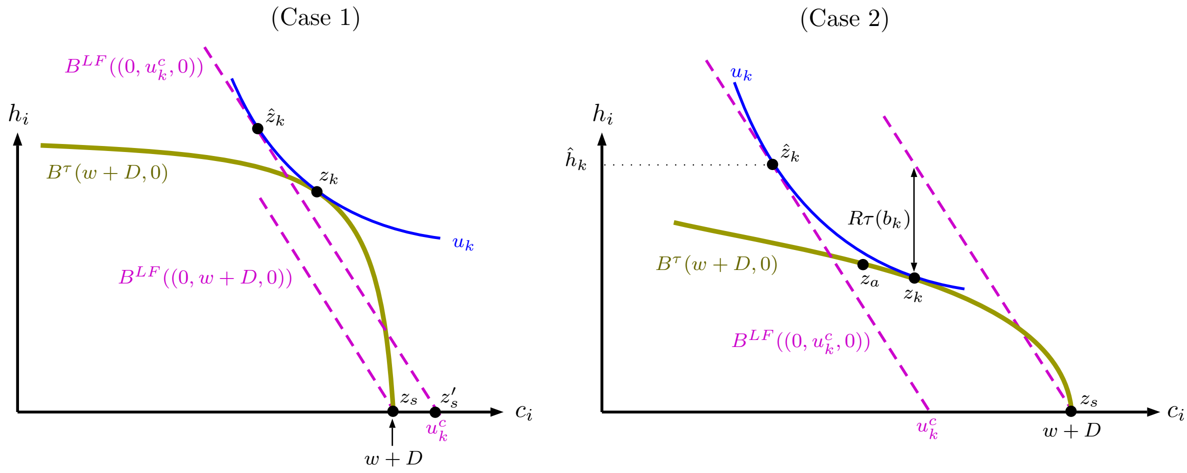


Figure 5: Constructions used in the proof of Lemma 1.

Since $z_k \notin B^{LF}((0, u_k^c, 0))$ but $u^c(z_k, u_k) = u_k^c$, this implies that for some bundle $\hat{z}_k = (0, \hat{c}_k, \hat{h}_k) \in B^{LF}((0, u_k^c, 0))$ we have $u_k(\hat{z}_k) = u_k(z_k)$ (see Figure 5 Case 2). This implies that

$$\hat{z}_k \in \arg \max_{\tilde{z}_k \in B^\tau(w+D,0) \cup B^{LF}((0,u_k^c,0))} u_k(\tilde{z}_k).$$

But by definition of the most-altruistic preferences $u^a$, it means that $a$ would chose in $B^\tau(w + D, 0) \cup B^{LF}((0, u_k^c, 0))$ a bundle $\hat{z}_a = (0, \hat{c}_a, \hat{h}_a)$ such that $\hat{h}_a \geq \hat{h}_k$. By construction, we have $\hat{z}_a \in B^{LF}((0, u_k^c, 0))$ and $\hat{z}_a \notin B^\tau(w + D, 0)$. This shows that $u^c(z_a, u_a) \leq u^c(\hat{z}_a, u_a) = u_k^c$ and thus $u^c(z_a, u_a) \leq u^c(z_k, u_k)$, a contradiction. This concludes the proof of Lemma 1. ■

Let $\mu_a^c = u^c(z_a, u_a)$ denote the long-run equilibrium $c$-equivalent utility of individual $a$ with $g_a = 0$ and $u_a = u^a$ under scheme $(\tau, D)$. Let $b_a$ denote the equilibrium bequest left by individual $a$ under scheme $(\tau, D)$.

**Lemma 2.** *Consider any sustainable tax-demogrant scheme $(\tau, D)$ such that $D \geq 0$ and $-\tau$ is single-peaked. Under A1 and A2, if $\tau$ is monotonically increasing, then $(\tau, D)$ is dominated if $\mu_a^c < w + D$ and $b_a > 0$. Under A1, A2 and A3, $(\tau, D)$ is dominated if $\mu_a^c < w + D - \underline{b}$ and $b_a > \underline{b}$.*

*Proof.* Let $z = (g_i, c_i, h_i)_{i \in [0,1]} \in S$ denote the long-run equilibrium allocation associated to $(\tau, D)$ and let $(b_i)_{i \in [0,1]}$ be the long-run equilibrium profile of bequests left, i.e. $b_i = w + D + g_i - c_i$ for all $i \in [0, 1]$. Let $A \subset [0, 1]$ be the subset of altruistic individuals, i.e. $u_i \neq u^s$ for all $i \in A$.

There are two cases to consider. For each case, the proof procedes in three steps. In Step 1, we construct a particular tax-demogrant scheme $(\tau', D')$. In Step 2, we show that $(\tau', D')$ is sustainable if $(\tau, D)$ is sustainable. In Step 3, we show that the long-run equilibrium allocation $z' \in S$ associated to $(\tau', D')$ is preferred by $\boldsymbol{R}^{c\text{–}lex}$ over $z$.

**CASE 1**: for all bequest amount $b > 0$ there is a subset $J \subseteq A$ with $\mu(J) > 0$ such that $b_j < b$ for all $j \in J$.

*Step 1.* We construct a particular tax-demogrant scheme $(\tau', D')$. The construction of $\tau'$ is based on a particular bequest amount $\beta > 0$, whose definition depends on the type of $\tau$.

- If $\tau$ is *monotonically increasing*, then $\underline{b} = 0$ and we take any $\beta$ such that $0 < \beta < \min(b_a, w + D - \mu_a^c)$.
- If $-\tau$ is *positive peak*, then $\underline{b} > 0$ and we take $\beta = \underline{b}$.

For both types, we have $0 < \beta < \min(b_a, w + D - \mu_a^c)$.

Given $\beta$, we construct $(\tau', D')$ from a specific member of a parametric family of "truncated" tax-demogrant schemes $(\tau^\Delta, D - \Delta)$ with parameter $\Delta \in (0, \beta)$. The construction of $(\tau^\Delta, D - \Delta)$ is illustrated in Figure 6 for the case $\underline{b} = 0$ and in Figure 4 for the case $\underline{b} > 0$ (where $\beta = \underline{b}$). All members of this family linearly truncate the budget set $B^\tau(w + D + g_i, 0)$ for bequests smaller than $\beta$ and differ by their associated demogrant $D - \Delta$.[8] Formally, we define $\tau^\Delta$ as

$$\tau^\Delta(x) := \begin{cases} \tau\left(x + \Delta\right) - \Delta & \text{for all } x \geq \beta - \Delta \\ \frac{\tau(\beta) - \Delta}{\beta - \Delta} x & \text{for all } x \in [0, \beta - \Delta]. \end{cases} \tag{4}$$

The particularity of scheme $(\tau^\Delta, D - \Delta)$ is that any individual $i \in [0, 1]$ for whom $b_i \geq \beta$ choses the same bundle under both $(\tau^\Delta, D - \Delta)$ and $(\tau, D)$, i.e.

$$\arg \max_{\tilde{z}_i \in B^\tau(w + D + g_i, 0)} u_i(\tilde{z}_i) = \arg \max_{\tilde{z}_i \in B^{\tau^\Delta}(w + D - \Delta + g_i, 0)} u_i(\tilde{z}_i).$$

Let $z^\Delta = (g_i^\Delta, c_i^\Delta, h_i^\Delta)_{i \in [0,1]} \in S$ denote the long-run equilibrium allocation associated to $(\tau^\Delta, D - \Delta)$ and let $(b_i^\Delta)_{i \in [0,1]}$ be the long-run equilibrium profile of bequests left, i.e. $b_i^\Delta = w + D - \Delta + g_i^\Delta - c_i^\Delta$ for all $i \in [0, 1]$. Recall that $R\beta - R\tau(\beta)$ is the amount inherited by the child of any $i \in [0, 1]$ for whom $b_i = \beta$. Consider the subset $J^\Delta \subseteq A$ for whom $h_j^\Delta < R\beta - R\tau(\beta)$ for all $j \in J^\Delta$.

We show that $\mu(J^\Delta) \to 0$ when $\Delta \to \beta$. The intuition for this statement is that the "truncated" budget set

$$B^{\tau^\Delta}\left(w + D - \Delta, 0\right)$$

has a kink at bundle $(0, w + D - \beta, R\beta - R\tau(\beta))$. Therefore, the larger is $\Delta$, the steeper is the slope of this truncated budget sets for small bequests (this slope tends to $-\infty$ when $\Delta \to \beta$), and the greater is the incentive to "bunch" at the kink for the "moderately" altruistic individuals who inherit nothing. More formally, for any altruistic preference $u \in U \backslash \{u^s\}$, there is a $\Delta < \beta$ such that

[8] Demogrant $D - \Delta$ need not be the maximal sustainable demogrant under tax $\tau^\Delta$.

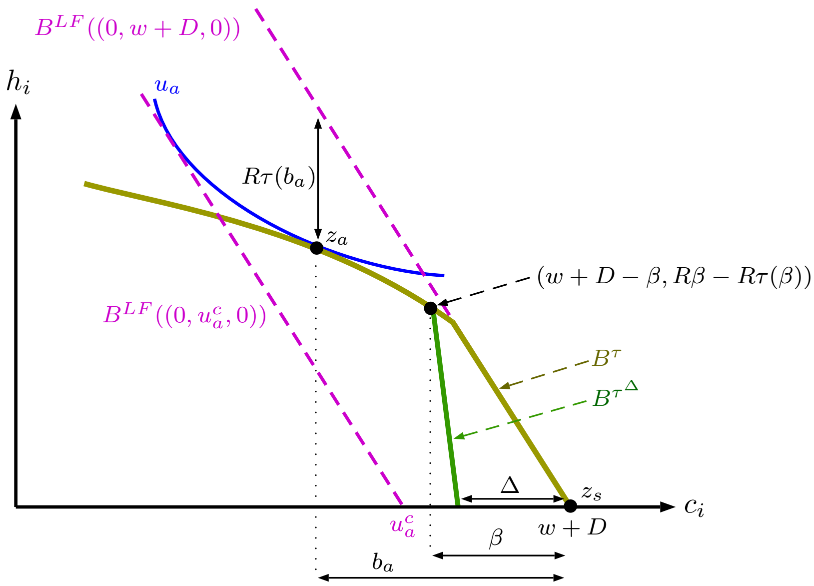


Figure 6: Construction of scheme $(\tau^\Delta, D-\Delta)$ for the case $\underline{b} = 0$.

for any $i \in [0,1]$ with $u_i = u$ and $g_i = 0$ we have $h_i^\Delta \geq R\beta - R\tau(\beta)$. This follows from A2, which requires that the altruistic preference $u_i$ is strictly monotonic in $h_i$ when $c_i \geq w$. The latter is guaranteed because the kink is located at bundle $(0, w+D-\beta, R\beta - R\tau(\beta))$ where $w + D - \beta \geq w$.[9] By the binormality of preferences, we also have $h_i^\Delta \geq R\beta - R\tau(\beta)$ for any $i \in [0,1]$ with an altruistic preference $u_i$ and $g_i > 0$, which yields the result.

We are now equipped for the definition of scheme $(\tau', D')$. This definition is based on the per-capita money amount $\beta\lambda_s$, which is saved by the government on the mass $\lambda_s$ of self-centered individuals when reducing the demogrant by an amount $\beta$. For some $\frac{\beta\lambda_s}{2} > 0$, consider the subset of altruistic individuals $J^{\frac{\beta\lambda_s}{2}} \subseteq A$ for whom $h_j < \frac{\beta\lambda_s}{2}$ for all $j \in J^{\frac{\beta\lambda_s}{2}}$. In words, all altruistic individuals in $J^{\frac{\beta\lambda_s}{2}}$ leave an inheritance smaller than half the per-capita amount saved on self-centered individuals. Under Case 1, we have $\mu(J^{\frac{\beta\lambda_s}{2}}) > 0$. Since $\mu(J^\Delta) \to 0$ when $\Delta \to \beta$, there is a value $\Delta^*$ with $\Delta^* > \frac{\beta}{2}$ such that[10]

- $\mu(J^{\Delta^*}) < \mu(J^{\frac{\beta\lambda_s}{2}})$, and
- $\tau^{\Delta^*}$ *subsidizes* bequests smaller than $\beta - \Delta^*$.[11]

In fact any $\Delta > \Delta^*$ also satisfies these two properties. We define scheme $(\tau', D')$

[9]Indeed, we assume that $\beta \leq w + D - u_a^c$ and we have $u_a^c \geq w$ (otherwise by A1 $\tau$ is dominated by Laissez-Faire).

[10]Recall that by definition of $\tau^\Delta$ we have $\Delta < \beta$.

[11]If the tax $\tau$ is positive peak, then $\tau^{\Delta^*}$ *subsidizes* bequests smaller than $\beta - \Delta^*$ for all $\Delta^* > 0$. In contrast, when the tax $\tau$ is monotonically increasing, $\tau^{\Delta^*}$ *subsidizes* small bequests when $\tau(\beta) < \Delta^*$.

as

$$(\tau', D') = \left(\tau^{\Delta^*}, D - \Delta^* + \frac{\beta\lambda_s}{2}\right),$$

whose construction is illustrated in Figure 7.

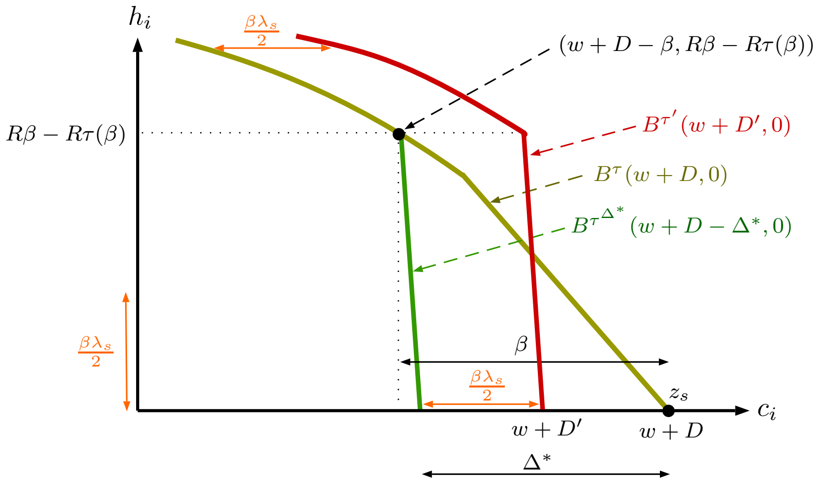


Figure 7: Construction of scheme $(\tau', D')$ for the case $\underline{b} = 0$.

*Step 2.* We show that $(\tau', D')$ is sustainable if $(\tau, D)$ is sustainable. Let $z' = (g'_i, c'_i, h'_i)_{i\in[0,1]} \in S$ denote the long-run equilibrium allocation associated to $(\tau', D')$ and let $(b'_i)_{i\in[0,1]}$ be the long-run equilibrium profile of bequests left, i.e. $b'_i = w + D' + g'_i - c'_i$ for all $i \in [0, 1]$. Let $(\hat{h}'_i)_{i\in[0,1]}$ be the profile obtained from $(h'_i)_{i\in[0,1]}$ when sorting dynasties by increasing order of $h'_i$, i.e. $\hat{h}'_j \leq \hat{h}'_k$ for any $j, k \in [0, 1]$ with $j < k$. Similarly, we use symbol "ˆ" to denote other bequest or inheritance profiles sorted in increasing order. Step 2 relies on the following Technical Claim.

*Technical Claim:* For any $i \in (\lambda_s + \mu(J^{\frac{\beta\lambda_s}{2}}), 1]$ we have $\hat{h}'_i \geq \hat{h}_i$ and $\hat{h}'_i \geq R\beta - R\tau(\beta)$.[12]

For $t \in \{0, 1, \dots\}$, we consider successive equilibrium allocations $z'_t = (g'_{it}, c'_{it}, h'_{it+1})_{i\in[0,1]}$ for which all $i \in [0, 1]$ chose in $B^{\tau'}(w + D' + g'_{it}, 0)$ and $(\hat{g}'_{it+1})_{i\in[0,1]} = (\hat{h}'_{it+1})_{i\in[0,1]}$, under an initial profile of inheritances $(g'_{i0})_{i\in[0,1]} = (h_i)_{i\in[0,1]}$, which corresponds to the long-run equilibrium profile associated to $(\tau, D)$. We show for all $t \in \{0, 1, \dots\}$ that

(1) $\hat{h}'_{it+1} + \frac{\beta\lambda_s}{2} \geq \hat{h}_i$ for all $i \in [0, 1]$,

(2) $\hat{h}'_{it+1} \geq \hat{h}_i$ and $\hat{h}'_{it+1} \geq R\beta - R\tau(\beta)$ for any $i \in (\lambda_s + \mu(J^{\frac{\beta\lambda_s}{2}}), 1]$.

[12]Index $i$ need not refer to the same dynasty in the two sorted distributions.

Observe that, at all $t$, (1) and (2) compare the sorted profile of inheritances left in $t$ to the sorted profile of inheritances left under the long-run equilibrium allocation $z$.

If (1) and (2) hold for all $t \geq 0$, then (2) holds as well for the profile $(h'_i)_{i\in[0,1]}$ associated to the long-run equilibrium allocation $z'$, because we assume that for any given tax-demogrant scheme, the economy converges over time to a unique long-run equilibrium allocation independent on the initial distribution of inheritances.

Consider first $t = 0$. In order to show that claims (1) and (2) hold for $t = 0$ when the tax-demogrant scheme is $(\tau', D')$, it is sufficient to show that claims (1) and (2) hold for $t = 0$ when the tax-demogrant scheme is $(\tau^{\Delta*}, D - \Delta^*)$ instead of $(\tau', D')$. Indeed, let $(h^{\Delta*}_{i1})_{i\in[0,1]}$ be the profile of inheritances obtained in period $t = 0$ if the tax-demogrant scheme is $(\tau^{\Delta*}, D - \Delta^*)$. Since $\tau' = \tau^{\Delta*}$ and $D' > D - \Delta^*$, the binormality of preferences implies that $h'_{i1} \geq h^{\Delta*}_{i1}$ for all $i \in [0, 1]$, which shows that it is indeed sufficient to prove these claims when the tax-demogrant scheme is $(\tau^{\Delta*}, D - \Delta^*)$.

Consider first claim (1) in $t = 0$, i.e.

$$\hat{h}^{\Delta*}_{i1} + \frac{\beta\lambda_s}{2} \geq \hat{h}_i \qquad \text{for all} \qquad i \in [0, 1]. \tag{5}$$

Consider the profile $(h_{i1})_{i\in[0,1]}$ of inheritances obtained in period $t = 0$ if the tax-demogrant scheme is $(\tau, D)$ instead of $(\tau^{\Delta*}, D-\Delta^*)$. As the initial profile $(g_{i0})_{i\in[0,1]} = (h_i)_{i\in[0,1]}$, we have $(\hat{h}_{i1})_{i\in[0,1]} = (\hat{h}_i)_{i\in[0,1]}$ because $z$ is the long-run equilibrium allocation associated to $(\tau, D)$. It is thus sufficient to show that

$$\hat{h}^{\Delta*}_{i1} + \frac{\beta\lambda_s}{2} \geq \hat{h}_{i1} \qquad \text{for all} \qquad i \in [0, 1] \tag{6}$$

for Eq. (5) to hold.

Since we have $h^{\Delta*}_{i1} = h_{i1} = 0$ for all $i \in [0, 1]$ for whom $u_i = u^s$, we can focus on the subset $A = \{i \in [0, 1] | u_i \neq u^s\}$ of altruistic individuals. We partition $A$ into three subgroups $A^1$, $A^2$ and $A^3$, respectively defined as

– $A^1 = \{i \in A | h_{i1} \geq R\beta - R\tau(\beta)\}$.
  By construction of $(\tau^{\Delta*}, D-\Delta^*)$, any $i \in A^1$ choses the *same* bundle in $B^{\tau}(w+D+g_{i0}, 0)$ and in $B^{\tau^{\Delta*}}(w+D-\Delta^*+g_{i0}, 0)$. This implies that $h^{\Delta*}_{i1} = h_{i1}$ for all $i \in A^1$.

– $A^2 = \{i \in A | h_{i1} < R\beta - R\tau(\beta)$ and $h^{\Delta*}_{i1} \geq R\beta - R\tau(\beta)\}$.
  By definition, we have $h^{\Delta*}_{i1} > h_{i1}$ for all $i \in A^2$.

– $A^3 = \{i \in A | h_{i1} < R\beta - R\tau(\beta)$ and $h^{\Delta*}_{i1} < R\beta - R\tau(\beta)\}$.
  By definition, any altruistic $j \in A^3$ leaves a smaller inheritance than $R\beta - R\tau(\beta)$, and thus a smaller inheritance than any $i \in A^1 \cup A^2$, i.e. $h^{\Delta*}_{j1} \leq h^{\Delta*}_{i1}$.

  We can assume without loss of generality that $\mu(A^3) \leq \mu(J^{\frac{\beta\lambda_s}{2}})$. If it is not the case, consider for the construction of $(\tau', D')$ a larger $\Delta \in (\Delta^*, \beta)$ for which we have $\mu(A^3) \leq \mu(J^{\frac{\beta\lambda_s}{2}})$. Such larger value exists by assumption A2.

By the definition of the above partition of $[0,1]$, we have

$$h_{i1}^{\Delta *} \geq h_{i1} \tag{7}$$

for all $i \in [0,1] \backslash A^3$,[13] but Eq. (7) may not hold for some $i \in A^3$. However, we have $\mu(A^3) \leq \mu(J^{\frac{\beta\lambda_s}{2}})$ and there is a subset of individuals of mass $\mu(J^{\frac{\beta\lambda_s}{2}})$ for whom $h_{i1} < \frac{\beta\lambda_s}{2}$. Therefore, even if $h_{i1}^{\Delta *} = 0$ for all $i \in A^3$, we have $h_{i1}^{\Delta *} + \frac{\beta\lambda_s}{2} \geq \frac{\beta\lambda_s}{2}$ for all $i \in A^3$,[14] which shows that (6) holds, and thus (5) holds.

We now turn to claim (2) for $t = 0$ if the tax-demogrant scheme is $(\tau^{\Delta *}, D - \Delta^*)$ instead of $(\tau', D')$, i.e.

$$\hat{h}_{i1}^{\Delta *} \geq \hat{h}_{i1} \text{ and } \hat{h}_{i1}^{\Delta *} \geq R\beta - R\tau(\beta) \text{ for all } i \in (\lambda_s + \mu(J^{\frac{\beta\lambda_s}{2}}), 1]. \tag{8}$$

By definition, if $i \in A^1 \cup A^2$ and $j \notin A^1 \cup A^2$ we have $h_{i1}^{\Delta *} \geq h_{j1}^{\Delta *}$. Also, as $A_2 \cap J^{\frac{\beta\lambda_s}{2}}$ may be non-empty, the mass of individuals in $A^1 \cup A^2$ is such that $\mu(A^1 \cup A^2) \geq 1 - \lambda_s - \mu(J^{\frac{\beta\lambda_s}{2}})$. Hence, it is sufficient that Eq. (8) holds for individuals in $A^1 \cup A^2$. As shown when defining these two subgroups, we have $h_{i1}^{\Delta *} = h_{i1} \geq R\beta - R\tau(\beta)$ for all $i \in A^1$ and $h_{i1}^{\Delta *} = R\beta - R\tau(\beta) > h_{i1}$ for all $i \in A^2$, which proves Eq. (8).

Together, we have shown claims (1) and (2) for $t = 0$. We next prove these claims for $t = 1$.

In $t = 1$, the profile of inheritances received under $(\tau', D')$ is such that $(\hat{g}'_{i1})_{i\in[0,1]} = (\hat{h}'_{i1})_{i\in[0,1]}$. By construction, since $\tau' = \tau^{\Delta *}$ and $D' = D - \Delta^* + \frac{\beta\lambda_s}{2}$, we have for all $i \in [0,1]$ that

$$B^{\tau^{\Delta^*}}\left(w + D - \Delta^* + g'_{i1} + \frac{\beta\lambda_s}{2}, 0\right) = B^{\tau'}\left(w + D' + g'_{i1}, 0\right),$$

as can be seen in Figure 7 (for the case $g'_{i1} = 0$). Therefore, the profile $(\hat{h}'_{i2})_{i\in[0,1]}$ obtained under $(\tau', D')$ for $(\hat{g}'_{i1})_{i\in[0,1]} = (\hat{h}'_{i1})_{i\in[0,1]}$ is the same as the profile $(\hat{h}_{i2}^{\Delta^*})_{i\in[0,1]}$ that would be obtained under $(\tau^{\Delta *}, D - \Delta^*)$ if the profile of inheritances received in $t = 1$ was instead $(\hat{h}'_{i1} + \frac{\beta\lambda_s}{2})_{i\in[0,1]}$. From claim (1) for $t = 0$, we have that $\hat{g}'_{i1} + \frac{\beta\lambda_s}{2} \geq \hat{g}'_{i0}$ for all $i \in [0,1]$. Therefore, by the binormality of preferences, the same reasoning implies again (1) and (2) for $t = 1$.

The same reasoning extends (1) and (2) to any $t \geq 2$, which concludes the proof of the Technical Claim.

Using the Technical Claim, we show that $(\tau', D')$ is sustainable if $(\tau, D)$ is sustainable. If $(\tau, D)$ is sustainable, then we have from the government's budget constraint that

$$0 \leq \int_{i\in[0,1]} (\tau(b_i) - D)\, di,$$

[13] We have shown that $h_{i1}^{\Delta *} = h_{i1}$ for all $i \in [0,1] \backslash A$, $h_{i1}^{\Delta *} = h_{i1}$ for all $i \in A^1$ and $h_{i1}^{\Delta *} > h_{i1}$ for all $i \in A^2$.

[14] In words, even if all individuals in $A^3$ leave no inheritances, there are enough altruistic individuals who, in the long-run equilibrium allocation $z$, leave inheritances smaller than the additional amount $\frac{\beta\lambda_s}{2}$ considered in claim (1).

where $(b_i)_{i\in[0,1]}$ is the long-run equilibrium profile of bequests left under $(\tau, D)$. In order to show that $(\tau', D')$ is sustainable, it is sufficient that[15]

$$\int_{i\in[0,1]} (\tau(b_i) - D)\, di \leq \int_{i\in[0,1]} (\tau'(b'_i) - D')\, di$$

where $(b'_i)_{i\in[0,1]}$ is long-run equilibrium profile of bequests left under $(\tau', D')$.

Recalling that the mass of self-centered individuals is $\lambda_s$, we have for all $i \in [0, \lambda_s]$ that $\hat{b}_i = \hat{b}'_i = 0$ and thus $\tau(\hat{b}_i) = \tau(\hat{b}'_i) = 0$ . Last inequality holds if the money saved by reducing the demogrant from $D$ to $D'$ is sufficient to cover the reduction in tax collected on altruistic individuals, i.e.

$$\int_{i\in(\lambda_s,1]} \tau(\hat{b}_i)di - \int_{i\in(\lambda_s,1]} \tau'(\hat{b}'_i)di \leq D - D'. \tag{9}$$

In the remainder of Step 2, we first show that Eq. (9) holds in a special case. Then, we build on this special case in order to show that Eq. (9) holds in general. To do that, using the Technical Claim, we show that the special case is in fact the worst-case scenario.

We now show that Eq. (9) holds for the special case for which $\hat{h}'_i = \hat{h}_i \geq R\beta - R\tau(\beta)$ for all $i \in (\lambda_s, 1]$. When $\hat{h}'_i \geq R\beta - R\tau(\beta)$, because $\tau' = \tau^{\Delta*}$ we have by the definition of $\tau^{\Delta}$ in Eq. (4) that $\tau'(\hat{b}'_i) = \tau\left(\hat{b}'_i + \Delta^*\right) - \Delta^*$. If $\hat{h}'_i = \hat{h}_i$, which is equivalent to $\hat{b}'_i - \tau'(\hat{b}'_i) = \hat{b}_i - \tau(\hat{b}_i)$, the definition of $\tau'$ implies that $\hat{b}'_i = \hat{b}_i - \Delta^*$ for all $i \in (\lambda_s, 1]$. In turn, this implies that $\tau'(\hat{b}'_i) = \tau(\hat{b}_i) - \Delta^*$. Replacing this expression in Eq. (9) yields

$$(1 - \lambda_s)\Delta^* \leq D - D'.$$

Since $D - D' = \Delta^* - \frac{\beta\lambda_s}{2}$, last inequality becomes $\Delta^* \geq \frac{\beta}{2}$, which holds as the construction of $\Delta^*$ is such that $\frac{\beta}{2} < \Delta^* < \beta$.

There remains to show that Eq. (9) holds in general if Eq. (9) holds for the special case for which $\hat{h}'_i = \hat{h}_i \geq R\beta - R\tau(\beta)$ for all $i \in (\lambda_s, 1]$. By the Technical Claim, we have $\hat{h}'_i \geq \hat{h}_i$ and $\hat{h}'_i \geq R\beta - R\tau(\beta)$ for all $i \in (\lambda_s + \mu(J^{\frac{\beta\lambda_s}{2}}), 1]$. From profiles $(\hat{h}_i)_{i\in(\lambda_s,1]}$ and $(\hat{h}'_i)_{i\in(\lambda_s,1]}$, we show it is possible to construct two alternative profiles $(\tilde{\hat{h}}_i)_{i\in(\lambda_s,1]}$ and $(\tilde{\hat{h}}'_i)_{i\in(\lambda_s,1]}$ that correspond to the special case, i.e. $\tilde{\hat{h}}'_i = \tilde{\hat{h}}_i \geq R\beta - R\tau(\beta)$ for all $i \in (\lambda_s, 1]$, and for which

$$\int_{i\in(\lambda_s,1]} \tau(\hat{b}_i)di - \int_{i\in(\lambda_s,1]} \tau'(\hat{b}'_i)di \leq \int_{i\in(\lambda_s,1]} \tau(\tilde{\hat{b}}_i)di - \int_{i\in(\lambda_s,1]} \tau'(\tilde{\hat{b}}'_i)di, \tag{10}$$

where $\tilde{\hat{h}}_i = R\tilde{\hat{b}}_i - R\tau(\tilde{\hat{b}}_i)$ and $\tilde{\hat{h}}'_i = R\tilde{\hat{b}}'_i - R\tau'(\tilde{\hat{b}}'_i)$ for all $i \in (\lambda_s, 1]$. If Eq. (10) holds, then Eq. (9) holds for $(\hat{b}_i)_{i\in(\lambda_s,1]}$ and $(\hat{b}'_i)_{i\in(\lambda_s,1]}$ because the inheritance profiles associated to $(\tilde{\hat{b}}_i)_{i\in(\lambda_s,1]}$ and $(\tilde{\hat{b}}'_i)_{i\in(\lambda_s,1]}$ correspond to the special case as $\tilde{\hat{h}}'_i = \tilde{\hat{h}}_i \geq R\beta - R\tau(\beta)$ for all $i \in (\lambda_s, 1]$.

[15] Recall that the sustainability of a scheme $(\tau, D)$ relates only to the government budget constraint under its associated long-run equilibrium allocation. Indeed, a long-run equilibrium allocation is by definition a steady-state allocation, implying that the profile of inheritances left corresponds to the profile of inheritances received.

We construct $(\tilde{\hat{h}}_i)_{i\in(\lambda_s,1]}$ from $(\hat{h}_i)_{i\in(\lambda_s,1]}$ as follows (see Figure 8.a for an illustration for the case of a positive peak tax $\tau$, for which $\tau(\beta) = 0$):

$$\tilde{\hat{h}}_j = R\beta - R\tau(\beta) \qquad \text{for all } j \in (\lambda_s, \lambda_s + \mu(J^{\frac{\beta\lambda_s}{2}})],$$
$$\tilde{\hat{h}}_i = \hat{h}_i \qquad \text{for all } i \in (\lambda_s + \mu(J^{\frac{\beta\lambda_s}{2}}), 1],$$

and construct $(\tilde{\hat{h}}'_i)_{i\in(\lambda_s,1]}$ from $(\hat{h}_i)_{i\in(\lambda_s,1]}$ and $(\hat{h}'_i)_{i\in(\lambda_s,1]}$ as follows (see Figure 8.b for an illustration for the case of a positive peak tax $\tau$, for which $\tau(\beta) = 0$):

$$\tilde{\hat{h}}'_j = R\beta - R\tau(\beta) \qquad \text{for all } j \in (\lambda_s, \lambda_s + \mu'],$$
$$\tilde{\hat{h}}'_i = \tilde{\hat{h}}_i \qquad \text{for all } i \in (\lambda_s + \mu', 1].$$

In words, profiles $(\tilde{\hat{h}}_i)_{i\in(\lambda_s,1]}$ and $(\tilde{\hat{h}}'_i)_{i\in(\lambda_s,1]}$ are constructed by replacing the inheritances $\hat{h}_j$ and $\hat{h}'_j$ that are smaller than $R\beta - R\tau(\beta)$ by $R\beta - R\tau(\beta)$, and then replacing inheritances $\hat{h}'_i$ that are larger than $\hat{h}_i$ by $\hat{h}_i$.

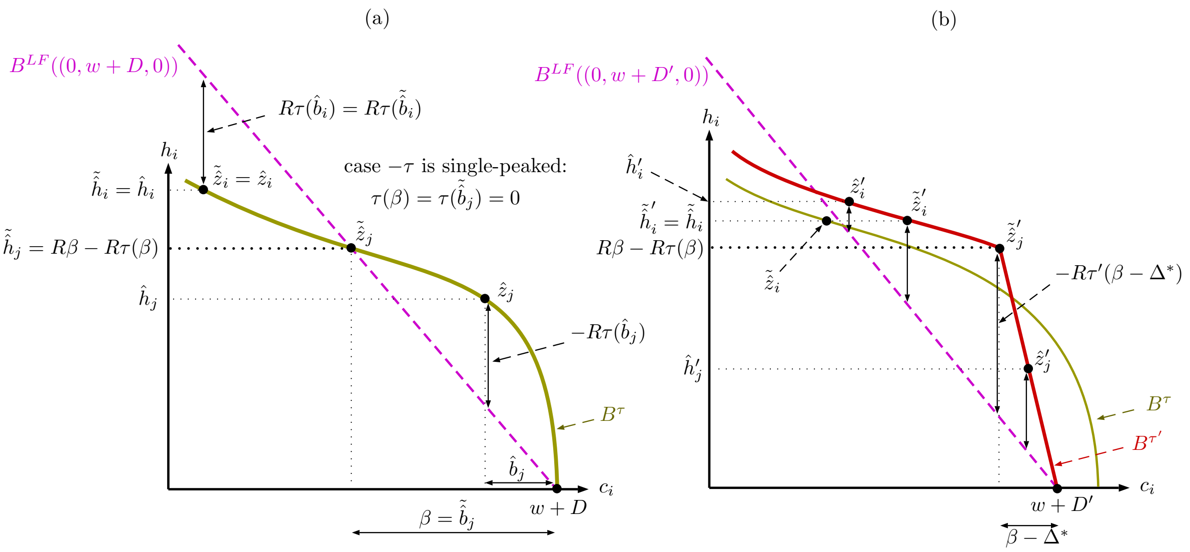


Figure 8: (a) Construction of profile $(\tilde{\hat{h}}_i)_{i\in(\lambda_s,1]}$, where $j \in (\lambda_s, \lambda_s + \mu(J^{\frac{\beta\lambda_s}{2}})]$ and $i \in (\lambda_s + \mu(J^{\frac{\beta\lambda_s}{2}}), 1]$. (b) Construction of profile $(\tilde{\hat{h}}'_i)_{i\in(\lambda_s,1]}$, where $j \in (\lambda_s, \lambda_s + \mu']$ and $i \in (\lambda_s + \mu', 1]$. Case of a positive peak tax $\tau$, for which $\tau(\beta) = 0$.

There remains to show that Eq. (10) holds. First, we show that

$$\int_{i\in(\lambda_s,1]} \tau(\hat{b}_i)di \leq \int_{i\in(\lambda_s,1]} \tau(\tilde{\hat{b}}_i)di. \tag{11}$$

For all $i \in (\lambda_s + \mu(J^{\frac{\beta\lambda_s}{2}}), 1]$, we have $\hat{h}_i = \tilde{\hat{h}}_i \geq R\beta - R\tau(\beta)$ implying that $\tau(\hat{b}_i) = \tau(\tilde{\hat{b}}_i)$. For all $j \in (\lambda_s, \lambda_s + \mu(J^{\frac{\beta\lambda_s}{2}})]$, we have $\hat{h}_j < R\beta - R\tau(\beta)$ and

$\tilde{\hat{h}}_j = R\beta - R\tau(\beta)$, and we show that $\tau(\hat{b}_j) \leq \tau(\tilde{\hat{b}}_j)$. This is obvious if $\tau$ is monotonically increasing because $\hat{b}_j < \tilde{\hat{b}}_j = \beta$. If $\tau$ is positive peak (the case illustrated in Figure 8.a), then our construction is such that $\beta = \underline{b}$. As $-\tau$ is single-peaked, this implies that $\tau(\hat{b}_j) \leq 0$ whereas $\tau(\beta) = 0$. Therefore Eq. (11) holds.

Second, we show that

$$\int_{i \in (\lambda_s, 1]} \tau'(\hat{b}'_i) di \geq \int_{i \in (\lambda_s, 1]} \tau'(\tilde{\hat{b}}'_i) di. \tag{12}$$

For all $i \in (\lambda_s + \mu', 1]$, we have $\hat{h}'_i \geq \tilde{\hat{h}}'_i \geq R\beta - R\tau(\beta)$ and we show that $\tau'(\hat{b}'_i) \geq \tau'(\tilde{\hat{b}}'_i)$ (see illustration in Figure 8.b, where $-\tau'(\hat{b}'_i) \leq -\tau'(\tilde{\hat{b}}'_i)$). The construction of $\tau'$ from $\tau$ is such that $-\tau'$ is single-peaked when $-\tau$ is single-peaked. Also, as $\hat{h}'_i \geq \tilde{\hat{h}}'_i \geq R\beta - R\tau(\beta)$ we have that $\hat{b}'_i \geq \tilde{\hat{b}}'_i \geq \beta - \Delta^*$. For bequest amounts larger than $\beta - \Delta^*$, $\tau'$ is monotonically increasing in bequest, which implies that $\tau'(\hat{b}'_i) \geq \tau'(\tilde{\hat{b}}'_i)$. For all $j \in (\lambda_s, \lambda_s + \mu']$, we have $\hat{h}'_j < R\beta - R\tau(\beta)$ and $\tilde{\hat{h}}'_j = R\beta - R\tau(\beta)$, and we show that $\tau'(\hat{b}'_j) \geq \tau'(\tilde{\hat{b}}'_j)$. For such $j$, we have thus $\hat{b}'_j < \tilde{\hat{b}}'_j = \beta - \Delta^*$. As $\tau' = \tau^{\Delta^*}$ and $\hat{b}'_j < \beta - \Delta^*$, we have $\tau'(\hat{b}'_j) < 0$. What is more, the construction of $\tau'$ is such that $\tau'(x') < \tau'(x)$ for all $0 \leq x < x' \leq \beta - \Delta^*$, i.e. the subsidy received under $\tau'$ is increasing in the bequest left, when the bequest is smaller than $\beta - \Delta^*$. Therefore Eq. (12) holds.

Eq. (11) and Eq. (12) together imply that (10) holds, which concludes Step 2.

*Step 3.* We show that allocation $z'$ is preferred by $\boldsymbol{R}^{c\text{–}lex}$ to allocation $z$.

By assumption A1, there is a positive mass of individuals $a$ and a positive mass of individuals $s$. The construction of $\tau'$ from $\tau$ is such that $-\tau'$ is single-peaked when $-\tau$ is single-peaked. By Lemma 1, under both $z'$ and $z$, either $a$ or $s$ are among the worst-offs. Individual $a$ is among the worst-offs under $z$ because $\mu^c_a = u^c(z_a, u_a) < w + D = u^c(z_s, u_s)$, as illustrated in Figure 6.

There remains to show that $\mu^c_a < u^c(z'_a, u_a)$ and $\mu^c_a < u^c(z'_s, u_s)$. We have $\mu^c_a < u^c(z'_s, u_s)$ because $u^c(z'_s, u_s) = w + D' > w + D - \beta$ and $\beta \leq w + D - u^c_a$.[16] Finally, we show that $\mu^c_a < u^c(z'_a, u_a)$. We have selected $\beta$ such that $\beta < b_a$.[17] When $\beta < b_a$, the construction of $\tau^{\Delta^*}$ implies that $z^{\Delta^*}_a = z_a$, where $z^{\Delta^*}_a$ is the equilibrium bundle of $a$ under scheme $(\tau^{\Delta^*}, D - \Delta^*)$. Since $\tau' = \tau^{\Delta^*}$ but $D' > D - \Delta^*$, bundle $z_a$ lies in the *interior* of[18]

$$B^{\tau'}(w + D', 0),$$

which is $a$'s budget set under $(\tau', D')$, as illustrated in Figure 7. This implies that $u_a(z_a) < u_a(z'_a)$, hence $\mu^c_a < u^c(z'_a, u_a)$.

**CASE 2**: there exists a bequest amount $b^*$ with $0 < b^* < \min(b_a, w + D - \mu^c_a)$ such that for all $J \subseteq A$ with $\mu(J) > 0$ we have $b_j \geq b^*$ for all $j \in J$.

[16] We have $w + D' > w + D - \beta$ because $D' = D - \Delta^* + \frac{\beta\lambda_s}{2}$ and $\Delta^* \in (\beta/2, \beta)$ and $\lambda_s \in (0, 1)$.

[17] If $\tau$ is monotonically increasing, then $\underline{b} = 0$ and this case is such that $\beta < b_a$. If $\tau$ is positive peak, then $\underline{b} > 0$ and this case is such that $\beta = \underline{b}$ and $b_a > \underline{b}$.

[18] As $D' > D - \Delta^*$, all bundles in $B^{\tau^{\Delta^*}}(w + D - \Delta^*, 0)$ lie in the interior of $B^{\tau'}(w + D', 0)$.

*Step 1.* We construct $(\tau', D')$ from a particular tax-demogrant scheme $(\tau'', D'')$. Take $D'' = D - b^*/2$. The tax $\tau''$ is constructed from $\tau$ by linearly truncating the budget set $B^\tau(w + D + g_i, 0)$ for bequests smaller than $b^*$ (as illustrated in Figure 9). Formally, we define $\tau''$ as

$$\tau''(x) := \begin{cases} \tau\left(x + \frac{b^*}{2}\right) - \frac{b^*}{2} & \text{for all } x \geq b^*/2 \\ \frac{\tau(b^*) - b^*/2}{b^*/2} x & \text{for all } x \in [0, b^*/2]. \end{cases}$$

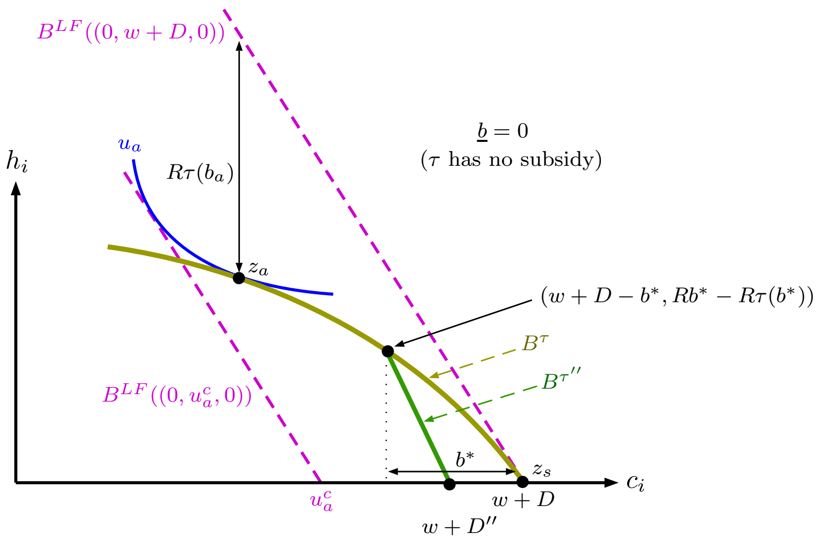


Figure 9: The tax-demogrant scheme $(\tau, D)$ is dominated because the tax function $\tau$ taxes small bequests. Individual $a$ is the worst-off because $u^c(z_a, u_a) = u_a^c$. The sustainable scheme $(\tau'', D'')$ has a smaller demogrant, does not affect $u^c(z_a, u_a)$ and leaves money on the table.

The particularity of scheme $(\tau'', D'')$ is that any individual $i \in [0,1]$ for whom $b_i \geq b^*$ choses the same bundle under both $(\tau'', D'')$ and $(\tau, D)$, i.e.

$$\arg \max_{\tilde{z}_i \in B^\tau(w+D+g_i,0)} u_i(\tilde{z}_i) = \arg \max_{\tilde{z}_i \in B^{\tau''}(w+D''+g_i,0)} u_i(\tilde{z}_i).$$

Let $z'' = (g_i'', c_i'', h_i'')_{i\in[0,1]} \in S$ denote the long-run equilibrium allocation associated to $(\tau'', D'')$ and let $(b_i'')_{i\in[0,1]}$ be the long-run equilibrium profile of bequests left, i.e. $b_i'' = w + D'' + g_i'' - c_i''$ for all $i \in [0,1]$. Case 2 is such that the profile $(\hat{h}_i'')_{i\in[0,1]} = (\hat{h}_i)_{i\in[0,1]}$ because for all $J \subseteq A$ with $\mu(J) > 0$ we have $b_j \geq b^*$ for all $j \in J$. This implies for all $i \in (\lambda_s, 1]$ that $\hat{b}_i'' = \hat{b}_i - b^*/2$ and thus $\tau''(\hat{b}_i'') = \tau(\hat{b}_i) - b^*/2$. As $(\tau, D)$ is sustainable, we have that $(\tau'', D'')$ leaves on the table an amount at least $\lambda_s \frac{b^*}{2}$, i.e.

$$\frac{\lambda_s b^*}{2} \leq \int_{i\in[0,1]} (\tau''(b_i'') - D'')\, di.$$

If $\tau$ is positive peak, then by assumption A3 there exists a sustainable scheme $(\tau', D')$ with $\tau' = \tau''$ and $D' > D''$. If $\tau$ is monotonically increasing, then A3 is not assumed and we define scheme $(\tau', D')$ as

$$(\tau', D') = \left(\tau'', D'' + \frac{b^* \lambda_s}{4}\right).$$

*Step 2.* We show that $(\tau', D')$ is sustainable if $(\tau, D)$ is sustainable. We have already shown it using A3 in Step 1 in the case for which $\tau$ is positive peak. In the case for which $\tau$ is monotonically increasing, then a simplified version of the argument used in Step 2 of Case 1 shows that $(\tau', D')$ is sustainable. The argument can be simplified because all altruistic individuals leave a bequest larger than $b^*$, implying that the partition of $A$ as $A = A^1 \cup A^2 \cup A^3$ is such that $A^2 = A^3 = \varnothing$. We do not repeat this argument.

*Step 3.* The long-run equilibrium allocation $z'$ associated to $(\tau', D')$ is preferred by $\boldsymbol{R}^{c\text{–}lex}$ to allocation $z$. The argument is the same as the argument used in Step 3 of Case 1. We do not repeat this argument. This concludes the proof of Lemma 2. ■

First, we prove claim (i) of Proposition 2. Assume to the contrary that $\tau$ is monotonically increasing but $\tau$ does not provide an exemption up to $b_a^{LF}(w)$, i.e. $\tau(b_a^{LF}(w)) > 0$. Under this contradiction assumption, we show that scheme $(\tau, D)$ is not optimal whatever the value of $D$. As any scheme $(\tau, D)$ with $D < 0$ is not optimal (Proposition 1), we consider any $(\tau, D)$ with $D \geq 0$.

As $\tau$ is monotonically increasing, we have $\underline{b} = 0$ and either $\tau(b_a) > 0$ or $\tau(b_a) = 0$. (Recall that $b_a$ denotes the equilibrium bequest left by individual $a$ with $g_a = 0$ and $u_a = u^a$.) If $\tau(b_a) > 0$, then Eq. (3) is violated and $(\tau, D)$ is not optimal by Proposition 2 (ii), whose proof is given below. So assume that $\tau(b_a) = 0$, which implies that $\tau(x) = 0$ for all $x \in [0, b_a]$ because $\tau$ is monotonically increasing. If $b_a \geq b_a^{LF}(w)$, then we have $\tau(b_a) > 0$ because $\tau(b_a^{LF}(w)) > 0$ and $\tau$ is monotonically increasing, a contradiction to our assumption that $\tau(b_a) = 0$.

There remains the case for which $b_a < b_a^{LF}(w)$ and $\tau(b_a) = 0$. First, we show that any optimal $(\tau, D)$ has $b_a > 0$. If $b_a = 0$ under $(\tau, D)$ with $D \geq 0$, then we can show that the economy converges to a long-run equilibrium allocation for which all inheritances are zero, i.e. $D = 0$. Indeed, if $b_a = 0$ under a scheme $(\tau, D)$ with $D \geq 0$, the binormality of preferences implies that $a$ leaves no bequest under scheme $(\tau, 0)$. Therefore, any dynasty $i$ with a member $it'$ such that $u_{it'} = u^s$ leaves no bequest for all $it$ with $t \geq t'$. As there is in each generation a mass $\lambda_s$ of individuals $i \in [0, 1]$ with $u_i = u^s$, and preferences are drawn at random in each generation, all dynasties have a member $it'$ such that $u_{it'} = u^s$ for some $t' \leq t$ when $t$ is sufficiently large. Therefore all inheritances are zero in the long-run equilibrium allocation, which implies that the largest sustainable demogrant is $D = 0$ when $b_a = 0$. We show that $(\tau, D = 0)$ is dominated by Laissez-Faire when $b_a = 0$. Under Laissez-Faire, the sustainable demogrant is also zero. The equilibrium bundle of $a$ under $(\tau, D = 0)$ is $z_a = (0, w, 0)$ because $b_a = 0$, whereas it is $z_a^{LF} = (0, w - b_a^{LF}(w), R b_a^{LF}(w))$ with $b_a^{LF}(w) > 0$ under Laissez-Faire. As illustrated in Figure 10.a, we have $u_a(z_a) <$

$u_a(z_a^{LF})$ because $z_a \in B^{LF}((0,w,0))$ but

$$z_a \neq z_a^{LF} = \arg\max_{\tilde{z}_a \in B^{LF}((0,w,0))} u_a(\tilde{z}_a).$$

Under Laissez-Faire, any individual $i \in [0,1]$ allocates her lifetime resources freely in the laissez-faire budget $B^{LF}((0, w+g_i, 0))$, thus we have $u^c(z_i^{LF}, u_i) = w + g_i$. This shows that $u^c(z_i^{LF}, u_i) \geq u^c(z_a^{LF}, u_a)$ for all $i \in [0,1]$ because $u^c(z_a^{LF}, u_a) = w$ as $g_a = 0$. Now, since $u_a(z_a^{LF}) > u_a(z_a)$, we have $u^c(z_a^{LF}, u_a) > u^c(z_a, u_a)$, implying that $u^c(z_i^{LF}, u_i) > u^c(z_a, u_a)$ for all $i \in [0,1]$. By A1, there is mass of individuals $a$ with $g_a = 0$ and $u_a = u^a$, showing that $(\tau, D = 0)$ is dominated by Laissez-Faire, i.e. not optimal.

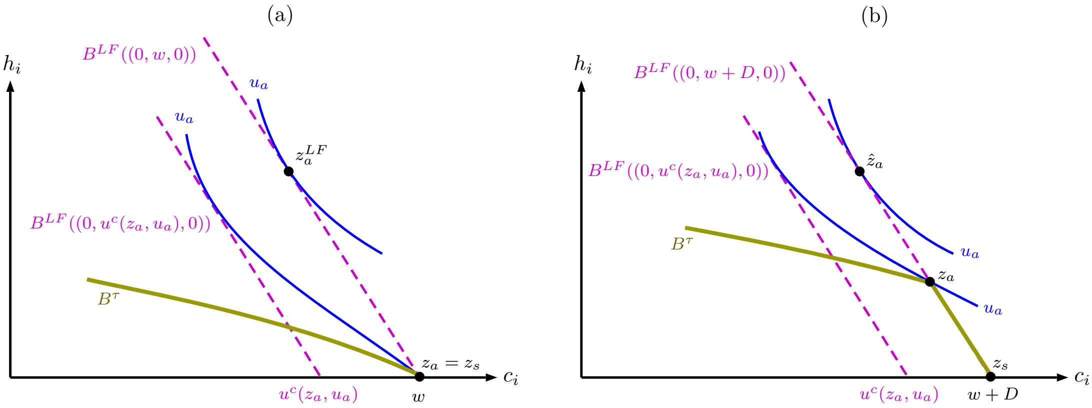


Figure 10: (a) $(\tau, D = 0)$ is dominated by Laissez-Faire when $b_a = 0$. (b) $u^c(z_a, u_a) < w + D$ when $0 < b_a < b_a^{LF}(w)$.

Now, for the case $0 < b_a < b_a^{LF}(w)$, the binormality of preferences implies that $b_a^{LF}(w) \leq b_a^{LF}(w+D)$ and thus $b_a < b_a^{LF}(w+D)$. In words, $a$ would increase her bequest if the exemption proposed by $\tau$ was larger. As illustrated in Figure 10.b, because $z_a \in B^{LF}((0, w+D, 0))$ but

$$z_a \neq \hat{z}_a = \arg\max_{\tilde{z}_a \in B^{LF}((0,w+D,0))} u_a(\tilde{z}_a),$$

we have $u_a(z_a) < u_a(\hat{z}_a)$.

As $u^c(\hat{z}_a, u_a) = w + D$, this implies that $u^c(z_a, u_a) < w + D$. As $\underline{b} = 0$ and $b_a > 0$, we have $b_a > \underline{b}$, and Lemma 2 implies that $(\tau, D)$ is not optimal.

Second, we prove claim (ii) of Proposition 2. We show that, if $\tau(b_a) > \underline{b}$, then scheme $(\tau, D)$ is not optimal whatever the value of $D$. As any scheme $(\tau, D)$ with $D < 0$ is not optimal (Proposition 1), we consider any $(\tau, D)$ with $D \geq 0$. As any sustainable $(\tau, D)$ for which $D < D^{max}$ is dominated by $(\tau, D^{max})$, and thus not optimal, we can focus on $D = D^{max}$. We show that the preconditions for Lemma 2 are all met, which implies that $(\tau, D)$ is not optimal.

By definition of $\underline{b}$ we have $\tau(\underline{b}) = 0$. Since $\tau(b_a) > 0$, we have $b_a > \underline{b}$ because $-\tau$ is single-peaked, implying that $\tau$ is monotonically increasing in $x$ for all $x \geq \underline{b}$.

Any individual $a$ with $g_a = 0$ and $u_a = u^a$ choses the equilibrium bundle $z_a = (0, w + D - b_a, Rb_a - R\tau(b_a))$ in the budget set $B^\tau(w + D, 0)$. Bundle $z_a$ is on the frontier of the Laissez-Faire budget set $B^{LF}((0, w + D - \tau(b_a), 0))$, which implies that $u^c(z_a, u_a) \leq w + D - \tau(b_a)$. As $\tau(b_a) > \underline{b}$, this implies that $u^c(z_a, u_a) < w + D - \underline{b}$.

Together, we have $D \geq 0$ and we have shown $\mu_a^c < w + D - \underline{b}$ and $b_a > \underline{b}$. Therefore, Lemma 2 implies that $(\tau, D)$ is not optimal.

## 6.3 Proof of Proposition 3

The following axiom is the well-known Separability axiom, according to which agents who are assigned identical bundles in two allocations should not matter for the social ranking between these two allocations. The idea that they should not matter is captured by the requirement that the social ranking remain the same if the preferences and bundles assigned to these agents change in such a way that the bundles assigned to these agents remain identical in the two allocations.

**Axiom 4** (Separability)**.**
*For all economy* $u \in \mathcal{U}$*, steady-state allocations* $z = (g_i, c_i, h_i)_{i\in[0,1]}$, $z' = (g'_i, c'_i, h'_i)_{i\in[0,1]}$, $z'' = (g''_i, c''_i, h''_i)_{i\in[0,1]}$, $z''' = (g'''_i, c'''_i, h'''_i)_{i\in[0,1]} \in S$*, subset of individuals* $J \in M[0,1]$*, if*

- *for all* $j \in J$*:* $z_j = z''_j$ *and* $z'_j = z'''_j$*,*
- *for all* $j \in [0,1] \setminus J$*:* $u_j = u'_j$*,* $z_j = z'_j$ *and* $z''_j = z'''_j$*,*

*then* $z\ \boldsymbol{R}(u)\ z''$ *if and only if* $z'\ \boldsymbol{R}(u')\ z'''$*.*

The bite of this axiom is that it allows us to modify the economy in such a way that sets of agents of positive measure have the same preferences, which is unlikely in a generic economy, whereas it is crucial to allow us to use Compensation (see Step 1 in the proof below). We now state and prove the following result, which justifies using SWF $\boldsymbol{R}^{c\text{-}lex}$.

**Proposition 3.** *If a SWF* $(R)$ *satisfies axioms Pareto, Compensation, Responsibility and Separability, then for all* $u \in \mathcal{U}$*,* $z = (g_i, c_i, h_i)_{i\in[0,1]}, z' = (g'_i, c'_i, h'_i)_{i\in[0,1]} \in S$*, if there exists* $J \in M[0,1]$ *such that* $\mu(J) > 0$ *and*

$$\sup_{j\in J} u^c(z_j, u_j) < \inf_{i\in[0,1]} u^c(z'_i, u_i)$$

*then*

$$z'\ \boldsymbol{P}(u)\ z.$$

*Proof.* This proof is reminiscent of similar proofs developed in models of labor income taxation in Fleurbaey and Maniquet (2006) and Fleurbaey and Maniquet (2007). The main differences are, first, that we deal here with economies with a continuum of agents, which makes some arguments longer, whereas all agents face the same prices (that is, the price of $h$ is equal to $R$), which allows us to simplify the proof.

The proof is divided in three steps. In the first step, we show that the combination of the four axioms implies a strengthening of the Responsibility axiom in which inequality aversion is infinite. In the second step, we show that the infinite inequality aversion is extended to $u^c(z_i, u_i)$. In the final step, we show that this allows us to derive the desired property.

*Step 1.* We begin by defining the following strengthening of Responsibility.

**Axiom 5** (Responsibility*)**.**
*For all economy $u \in \mathcal{U}$, steady-state allocations $z = (g_i, c_i, h_i)_{i\in[0,1]}$, $z' = (g'_i, c'_i, h'_i)_{i\in[0,1]} \in S$, subsets of individuals $J, K \in M[0,1]$ such that $\mu(J) = \mu(K) > 0$, if there exists $\delta, \Delta > 0$ such that for all $j, q \in J$ and $k, \ell \in K$,*

- $z_i \in \max_{|u_i} B^{LF}(z_i)$, $\forall\, i \in \{j, q, k, \ell\}$,
- $z'_i \in \max_{|u_i} B^{LF}(z'_i)$, $\forall\, i \in \{j, q, k, \ell\}$,
- $y_j + \delta = y_q + \delta = y'_j = y'_q \leq y'_k = y'_\ell = y_k - \Delta = y_\ell - \Delta$,

*where*

$$y_i = c_i + \frac{h_i}{R}, y'_i = c'_i + \frac{h'_i}{R}, \forall\, i \in \{j, q, k, \ell\}$$

*and $z_i = z'_i$ for all $i \notin J \cup K$ then $z'$ $\boldsymbol{P}(u)$ $z$.*

With Responsibility*, we require strict social preference as soon as all agents in $J$ gain, even if their budget gain is arbitrarily small and the budget loss of members of $K$ is arbitrarily large. This is why Responsibility*, contrary to Responsibility, conveys an infinite inequality aversion.

We prove the following claim: If a SWF $(R)$ satisfies Pareto, Compensation, Responsibility and Separability, then it satisfies Responsibility*.

Let $u \in \mathcal{U}$, $z = (g_i, c_i, h_i)_{i\in[0,1]}$, $z' = (g'_i, c'_i, h'_i)_{i\in[0,1]} \in S$, and $J, K \in M[0,1]$ satisfy the conditions of Responsibility*. Let us assume, contrary to the claim, that $z$ $\mathbf{R}(u)$ $z'$. We can assume, without loss of generality, that $\mu(J) = \mu(K) \leq \frac{1}{4}$. If it is not the case, then the claim is proven by repeating this proof twice. Let $y_j, y'_j, y_k, y'_k \in \mathbb{R}_+$ be defined by

$$y_i = c_i + \frac{h_i}{R} \,\forall\, i \in \{j, k\},$$

so that $y_j < y'_j \leq y'_k < y_k$. By Pareto, we can assume, w.l.o.g., that $y'_j < y'_k$. Indeed, if it is not the case, then we can create $z'' = (g''_i, c''_i, h''_i)_{i\in[0,1]} \in S$ by replacing $z'_k = (g'_k, c'_k, h'_k)$ with $z''_k = (g''_k, c''_k, h''_k)$ such that

$$y'_k < y''_k = c''_k + \frac{h''_k}{R} < y_k$$

for all $k \in K$, so that $z''$ $\mathbf{P}(u)$ $z'$ and, by transitivity, $z''$ $\mathbf{P}(u)$ $z$ and continue the proof. So, we assume $y'_j < y'_k$. We can even assume, w.l.o.g., that

$$(y'_j - y_j) + (y_k - y'_k) < \frac{y_k - y_j}{2}.$$

Indeed, if it is not the case, then the claim is proven by repeating this proof the required number of times.[19]

[19]The fact that $y'_k > y'_j$ always allows us to construct sets $A$ and $B$ with $\bar{y}'_a < \bar{y}_a = \bar{y}_b < \bar{y}'_b$ and the proof below has to be replicated a finite number of times at least as large as $\frac{y'_j - y_j}{\bar{y}'_a - \bar{y}_a}$.

Let $u^* \in U$ and $\bar{z}_a = (\bar{g}_a, \bar{c}_a, \bar{h}_a), \bar{z}'_a = (\bar{g}'_a, \bar{c}'_a, \bar{h}'_a), \bar{z}''_a = (\bar{g}''_a, \bar{c}''_a, \bar{h}''_a), \bar{z}'''_a = (\bar{g}'''_a, \bar{c}'''_a, \bar{h}'''_a), \bar{z}_b = (\bar{g}_b, \bar{c}_b, \bar{h}_b), \bar{z}'_b = (\bar{g}'_b, \bar{c}'_b, \bar{h}'_b), \bar{z}''_b = (\bar{g}''_b, \bar{c}''_b, \bar{h}''_b), \bar{z}'''_b = (\bar{g}'''_b, \bar{c}'''_b, \bar{h}'''_b) \in X$, $\bar{y}_a, \bar{y}'_a, \bar{y}_b, \bar{y}'_b \in \mathbb{R}$ be such that

$$\bar{y}_i = \bar{c}_i + \frac{\bar{h}_i}{R}, \bar{y}'_i = \bar{c}'_i + \frac{\bar{h}'_i}{R}, \forall\, i \in \{a, b\}$$

$$\begin{aligned} \bar{y}_a &= \bar{y}_b \\ \bar{y}_a - \bar{y}'_a &= y'_j - y_j \\ \bar{y}'_b - \bar{y}_b &= y_k - y'_k \\ y'_j &\leq \bar{y}'_a \\ \bar{y}'_b &\leq y'_k \end{aligned}$$

$$\bar{z}_a \in \max{}_{|u^*} B^{LF}(\bar{z}_a),\ \bar{z}'_a \in \max{}_{|u^*} B^{LF}(\bar{z}'_a),$$
$$\bar{z}_b \in \max{}_{|u^*} B^{LF}(\bar{z}_b),\ \bar{z}'_b \in \max{}_{|u^*} B^{LF}(\bar{z}'_b)$$
$$\bar{c}''_a < \bar{c}''_b, \bar{h}''_a < \bar{h}''_b,$$
$$(\bar{c}'''_a, \bar{h}'''_a) = (\bar{c}'''_b, \bar{h}'''_b) = \frac{(\bar{c}''_b, \bar{h}''_b) + (\bar{c}''_a, \bar{h}''_a)}{2}$$

$$\begin{aligned} u^*(z_a) &= u^*(z'''_a) \\ u^*(z'_a) &= u^*(z''_a) \\ u^*(z'_b) &= u^*(z''_b). \end{aligned}$$

The construction of $u^*, \bar{z}_a, \bar{z}'_a, \bar{z}''_a, \bar{z}'''_a, \bar{z}_b, \bar{z}'_b, \bar{z}''_b, \bar{z}'''_b, \bar{y}_a, \bar{y}'_a, \bar{y}_b, \bar{y}'_b$ is illustrated in Fig. 11.

The intuition of the proof and the role of the axioms can be illustrated with the figure. We need to prove that the budget increase of an amount $\delta$ for agents $j$ and $q$ at the expense of a budget decrease of an amount $\Delta$ for agents $k$ and $\ell$, with $\Delta$ possibly much larger than $\delta$, is a social improvement. Separability allows us to modify the preferences and bundles of a sufficiently large number of agents and insert agents of type $a$ and $b$ in the economy. The design of their preferences is key: they are indifferent between a transfer of resources, represented by bundles $\bar{z}''_i$ and $\bar{z}'''_i$, $i \in \{a, b\}$, in which the beneficiary gets an amount equal to that left by the contributor, and a transfer of resources in which the beneficiary gets a different amount of resources, possible much smaller, than the one lost by the contributor, represented by bundles $\bar{z}_i$ and $\bar{z}'_i$, $i \in \{a, b\}$. Pareto forces us to be indifferent between these two sets of transfers. Transfers are calibrated in such a way that a sequence of transfers between agents $j$ and $a$ (using Responsibility), agents $a$ and $b$ (using Compensation) and then agents $b$ and $k$ (using Responsibility) allows us to reach the desired conclusion.

The following axiom, known in the literature as Pareto indifference, is a well-known consequence of Pareto.

**Axiom 6** (Pareto Indifference)**.**
*For all economy $u \in \mathcal{U}$ and steady-state allocations $z = (g_i, c_i, h_i)_{i\in[0,1]}, z' = (g'_i, c'_i, h'_i)_{i\in[0,1]} \in S$, if for all $i \in [0, 1]$*

$$u_i(c'_i, h'_i) = u_i(c_i, h_i)$$

*then $z'$ $\boldsymbol{I}(u)$ $z$.*

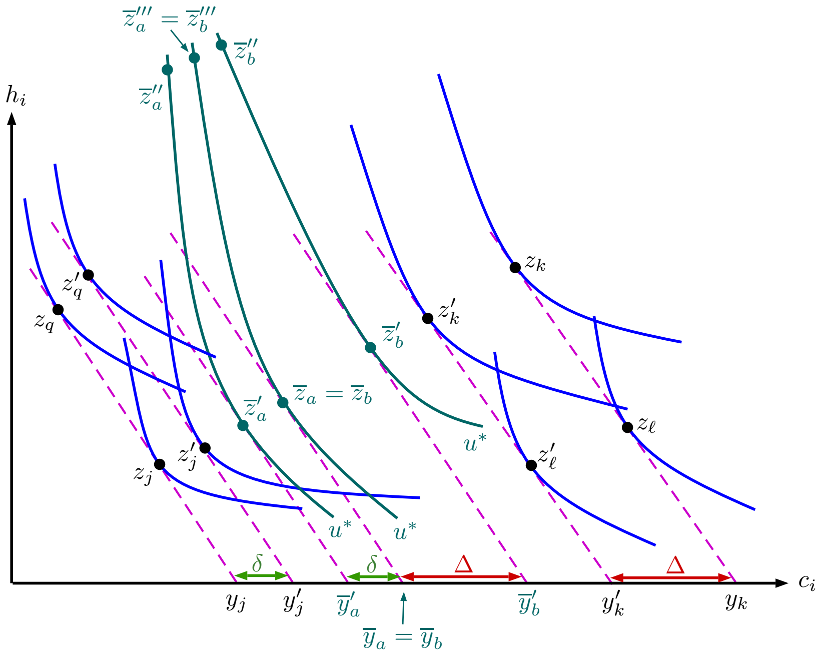


Figure 11: Construction of $u^*, \bar{z}_a, \bar{z}'_a, \bar{z}''_a, \bar{z}'''_a, \bar{z}_b, \bar{z}'_b, \bar{z}''_b, \bar{z}'''_b, \bar{y}_a, \bar{y}'_a, \bar{y}_b, \bar{y}'_b$.

Let $A, B \in M[0,1]$ be such that $\mu(A) = \mu(B) = \mu(J) = \mu(K)$, $A, B, J$ and $K$ are all disjoint. Since they are all disjoint, we have $(c_i, h_i) = (c'_i, h'_i)$ for all $i \in A \cup B$. Let $u' \in \mathcal{U}$ be defined by

$$\begin{aligned} u'_a &= u^*, \forall\, a \in A \\ u'_b &= u^*, \forall\, b \in B \\ u'_i &= u_i, \forall\, i \in [0,1] \setminus (A \cup B). \end{aligned}$$

Let allocations $z^1 = (g^1_i, c^1_i, h^1_i)_{i\in[0,1]}, z^2 = (g^2_i, c^2_i, h^2_i)_{i\in[0,1]} \in \mathbb{R}^{3[0,1]}_+$ be defined by

$$\begin{aligned} (c^1_a, h^1_a) = (c^2_a, h^2_a) &= (\bar{c}_a, \bar{h}_a), \forall\, a \in A \\ (c^1_b, h^1_b) = (c^2_b, h^2_b) &= (\bar{c}_b, \bar{h}_b), \forall\, b \in B, \end{aligned}$$

which implies $(c^1_a, h^1_a) = (c^1_b, h^1_b)$, and by

$$\begin{aligned} (c^1_i, h^1_i) &= (c_i, h_i), \forall\, i \in [0,1] \setminus (A \cup B), \\ (c^2_i, h^2_i) &= (c'_i, h'_i), \forall\, i \in [0,1] \setminus (A \cup B), \end{aligned}$$

and $g^1_i, g^2_i, i \in [0,1]$, are fixed so as to guarantee that $z^1, z^2 \in S$. By Separability,

$$z \ \mathbf{R}(u)\ z' \Leftrightarrow z^1 \ \mathbf{R}(u')\ z^2,$$

so that, by the premise of the argument, $z^1 \ \mathbf{R}(u')\ z^2$.

Let $z^3 = (g^3_i, c^3_i, h^3_i)_{i\in[0,1]} \in \mathbb{R}^{3[0,1]}_+$ be defined by

$$\begin{aligned}
(c^3_a, h^3_a) &= (\bar{c}'_a, \bar{h}'_a), \forall\, a \in A \\
(c^3_j, h^3_j) &= (c'_j, h'_j), \forall\, j \in J \\
(c^3_i, h^3_i) &= (c^1_i, h^1_i), \forall\, i \in [0,1] \setminus (A \cup J),
\end{aligned}$$

and $g^3_i, i \in [0,1]$, are fixed so as to guarantee that $z^3 \in S$. By Responsibility, $z^3\ \mathbf{P}(u')\ z^1$, so that, by transitivity, $z^3\ \mathbf{P}(u')\ z^2$.

Let $z^4 = (g^4_i, c^4_i, h^4_i)_{i\in[0,1]} \in \mathbb{R}^{3[0,1]}_+$ be defined by

$$\begin{aligned}
(c^4_b, h^4_b) &= (\bar{c}'_b, \bar{h}'_b), \forall\, b \in B \\
(c^4_k, h^4_k) &= (c'_k, h'_k), \forall\, k \in K \\
(c^4_i, h^4_i) &= (c^3_i, h^3_i), \forall\, i \in [0,1] \setminus (B \cup K),
\end{aligned}$$

and $g^4_i, i \in [0,1]$, are fixed so as to guarantee that $z^4 \in S$. By Responsibility, $z^4\ \mathbf{P}(u')\ z^3$, so that, by transitivity, $z^4\ \mathbf{P}(u')\ z^2$.

Let $z^5 = (g^5_i, c^5_i, h^5_i)_{i\in[0,1]} \in \mathbb{R}^{3[0,1]}_+$ be defined by

$$\begin{aligned}
(c^5_a, h^5_a) &= (\bar{c}''_a, \bar{h}''_a), \forall\, a \in A \\
(c^5_b, h^5_b) &= (\bar{c}''_b, \bar{h}''_b), \forall\, b \in B \\
(c^5_i, h^5_i) &= (c^4_i, h^4_i), \forall\, i \in [0,1] \setminus (A \cup B),
\end{aligned}$$

and $g^5_i, i \in [0,1]$, are fixed so as to guarantee that $z^5 \in S$. By Pareto Indifference, $z^5\ \mathbf{I}(u')\ z^4$, so that, by transitivity, $z^5\ \mathbf{P}(u')\ z^2$.

Let $z^6 = (g^6_i, c^6_i, h^6_i)_{i\in[0,1]} \in \mathbb{R}^{3[0,1]}_+$ be defined by

$$\begin{aligned}
(c^6_a, h^6_a) &= (\bar{c}'''_a, \bar{h}'''_a), \forall\, a \in A \\
(c^6_b, h^6_b) &= (\bar{c}'''_b, \bar{h}'''_b), \forall\, b \in B \\
(c^6_i, h^6_i) &= (c^5_i, h^5_i), \forall\, i \in [0,1] \setminus (A \cup B),
\end{aligned}$$

and $g^6_i, i \in [0,1]$, are fixed so as to guarantee that $z^6 \in S$. By Compensation, $z^6\ \mathbf{P}(u')\ z^5$, so that, by transitivity, $z^6\ \mathbf{P}(u')\ z^2$.

Let $z^7 = (g^7_i, c^7_i, h^7_i)_{i\in[0,1]} \in \mathbb{R}^{3[0,1]}_+$ be defined by

$$\begin{aligned}
(c^7_a, h^7_a) &= (\bar{c}_a, \bar{h}_a), \forall\, a \in A \\
(c^7_b, h^7_b) &= (\bar{c}_b, \bar{h}_b), \forall\, b \in B \\
(c^7_i, h^7_i) &= (c^6_i, h^6_i), \forall\, i \in [0,1] \setminus (A \cup B),
\end{aligned}$$

and $g^7_i, i \in [0,1]$, are fixed so as to guarantee that $z^7 \in S$. By Pareto Indifference, $z^7\ \mathbf{I}(u')\ z^6$, so that, by transitivity, $z^7\ \mathbf{P}(u')\ z^2$.

Let $z^8 = (g^8_i, c^8_i, h^8_i)_{i\in[0,1]}, z^9 = (g^9_i, c^9_i, h^9_i)_{i\in[0,1]} \in \mathbb{R}^{3[0,1]}_+$ be defined by

$$\begin{aligned}
z^8_a &= z_a, \forall\, a \in B \\
z^8_b &= z_b, \forall\, b \in B \\
z^8_i &= z^7_i, \forall\, i \in [0,1] \setminus (A \cup B),
\end{aligned}$$

and

$$\begin{aligned}
z^9_a &= z_a, \forall\, a \in B \\
z^9_b &= z_b, \forall\, b \in B \\
z^9_i &= z^2_i, \forall\, i \in [0,1] \setminus (A \cup B).
\end{aligned}$$

By Separability,

$$z^8 \; \mathbf{R}(u) \; z^9 \Leftrightarrow z^7 \; \mathbf{R}(u') \; z^2,$$

so that, by transitivity, $z^8 \; \mathbf{P}(u) \; z^9$. Finally, observe that $z^8 = z^9 = z'$, so that $z' \; \mathbf{P}(u) \; z'$, the desired contradiction.

*Step 2.* We now prove the following claim: If a SWF $(R)$ satisfies axioms Pareto, Compensation, Responsibility and Separability, then it satisfies the following property, which amounts to requiring an infinite aversion towards inequality in $u^c$: For all $u \in \mathcal{U}$, $z = (g_i, c_i, h_i)_{i\in[0,1]}$, $z' = (g'_i, c'_i, h'_i)_{i\in[0,1]} \in S$, $J, K \in M[0,1]$ such that $\mu(J) = \mu(K) > 0$, if for all $j \in J$ and $k \in K$,

- $u^c(z_j, u_j) < u^c(z'_j, u_j) < u^c(z'_k, u_k) < u^c(z_k, u_k)$,
- $\sup_{i\in J} u^c(z_i, u_i) < \inf_{i\in J} u^c(z'_i, u_i)$,

and $z_i = z'_i$ for all $i \notin J \cup K$ then $z' \; \mathbf{P}(u) \; z$.

Let $u \in \mathcal{U}$, $z = (g_i, c_i, h_i)_{i\in[0,1]}$, $z' = (g'_i, c'_i, h'_i)_{i\in[0,1]} \in S$, and $J, K \subseteq M[0,1]$ satisfy the conditions of this property. Let us assume, contrary to the claim, that $z \; \mathbf{R}(u) \; z'$.

By Pareto Indifference, we can assume, without loss of generality, that

$$z_j \in \max_{|u_j} B^{LF}(z_j),\; z'_j \in \max_{|u_j} B^{LF}(z'_j),$$
$$z_k \in \max_{|u_k} B^{LF}(z_k),\; z'_k \in \max_{|u_k} B^{LF}(z'_k).$$

Indeed, if it is not the case, then, by Pareto Indifference, we can replace bundles $z_j, z'_j$ and $z_k, z'_k$ by a bundle on the same indifference curve that is optimal in the corresponding budget.

By Pareto, we can assume that for all $j, q \in J$, $c_j + \frac{h_j}{R} = c_q + \frac{h_q}{R}$ and for all $k, \ell \in J$, $c_k + \frac{h_k}{R} = c_\ell + \frac{h_\ell}{R}$. Indeed, if it is not the case, we can replace each $z_j$ with $z''_j$ such that $u^c(z''_j, u_j) = \sup_{i\in J} u^c(z_i, u_i)$, each $z'_j$ with $z'''_j$ such that $u^c(z'''_j, u_j) = \inf_{i\in J} u^c(z_i, u_i)$, each $z_k$ with $z''_k$ such that $u^c(z''_k, u_k) = \sup_{i\in K} u^c(z_i, u_i)$, each $z'_k$ with $z'''_k$ such that $u^c(z'''_k, u_k) = \inf_{i\in K} u^c(z_i, u_i)$. By Pareto, $z'' \; \mathbf{P}(u) \; z$ and $z' \; \mathbf{P}(u) \; z'''$, so that $z'' \; \mathbf{P}(u) \; z'''$, and the proof continues.

By Responsibility*, $z' \; \mathbf{P}(u) \; z$, so that, by transitivity, $z' \; \mathbf{P}(u) \; z'$, the desired contradiction.

*Step 3.* We now prove the claim presented in the statement of the Proposition. Let $u \in \mathcal{U}$, $z = (g_i, c_i, h_i)_{i\in[0,1]}$, $z' = (g'_i, c'_i, h'_i)_{i\in[0,1]} \in S$, $J \in M[0,1]$ such that $\mu(J) > 0$ and

$$\sup_{j\in J} u^c(z_j, u_j) < \inf_{i\in[0,1]} u^c(z'_i, u_i).$$

Let us assume, contrary to the claim, that $z \; \boldsymbol{R}(u) \; z'$. Let $z'' = (g''_i, c''_i, h''_i)_{i\in[0,1]} \in S$ be such that $u^c(z''_i, u_i) = u$ for all $i \in [0,1]$ and

$$\sup_{j\in J} u^c(z_j, u_j) < u < \inf_{i\in[0,1]} u^c(z'_i, u_i).$$

Let $N \in \mathcal{N}$ be an integer such that $N\mu(J) > 1$. Let $J' \subseteq J$ be such that $\mu(J') = \frac{1-\mu(J)}{N}$. We can create a sequence $z^0, \ldots, z^n, \ldots, z^N$ such that $z^0_i = z_i$ for all $i \in [0,1] \setminus (J \setminus J')$, $u^c(z^0_j, u_j) = u$ for all $j \in J \setminus J'$, $z^N = z''$ and for each $n \in \{1, \ldots, N\}$, there exists a set $K^n \in M[0,1]$ such that $\mu(K^n) = \mu(J')$, $\cup_{n\in\{1,\ldots,N\}} K^n \cup J = [0,1]$,

$$\begin{aligned}
u^c(z^n_k, u_k) &= u, \forall\, k \in K^n \\
u^c(z^n_j, u_j) &= u^c(z^{n-1}_j, u_j) + \frac{1}{N}\left(u - u^c(z_j, u_j)\right), \forall\, j \in J' \\
u^c(z^n_j, u_j) &= u, \forall\, k \in J \setminus J' \\
z^n_i &= z^{n-1}_i, \forall\, i \in [0,1] \setminus (K^n \cup J').
\end{aligned}$$

By Pareto, $z^0 \; \boldsymbol{P}(u) \; z$. By the property proven in Step 3, $z^n \; \boldsymbol{P}(u) \; z^{n-1}$. By transitivity, $z'' \; \boldsymbol{P}(u) \; z$. By Pareto, $z' \; \boldsymbol{P}(u) \; z''$, so that $z' \; \boldsymbol{P}(u) \; z$, the desired contradiction.

■

# References


Atkinson, A. and Stiglitz, J. (1976). The design of tax structure: direct versus indirect taxation. *Journal of Public Economics*, 6(1):55–75.

Chamley, C. (1986). Optimal Taxation of Capital Income in General Equilibrium with Infinite Lives. *Econometrica*, 54(3):607–622.

Cremer, H. and Pestieau, P. (2006). Chapter 16 Wealth transfer taxation: a survey of the theoretical literature. *Handbook of the Economics of Giving, Altruism and Reciprocity*, 2(06):1107–1134.

Cremer, H., Pestieau, P., and Rochet, J.-c. (2001). Direct versus Indirect Taxation : The Design of the Tax Structure Revisited. *International Economic Review*, 42(3):781–799.

Cremer, H., Pestieau, P., and Rochet, J. C. (2003). Capital income taxation when inherited wealth is not observable. *Journal of Public Economics*, 87(11):2475–2490.

Farhi, E. and Werning, I. (2010). Progressive Estate Taxation. *The Quarterly Journal of Economics*, 125(2):635–673.

Fleurbaey, M., Leroux, M.-L., Pestieau, P., Ponthiere, G., and Zuber, S. (2018). Premature deaths, accidental bequests and fairness.

Fleurbaey, M. and Maniquet, F. (2006). Fair income tax. *The Review of Economic Studies*, 73(1):55–83.

Fleurbaey, M. and Maniquet, F. (2007). Help the low skilled or let the hardworking thrive? a study of fairness in optimal income taxation. *Journal of Public Economic Theory*, 9(3):467–500.

Fleurbaey, M. and Maniquet, F. (2018). Optimal Taxation Theory and Principles of Fairness. *Journal of Economic Literature*, 56(3):1029–1079.

Golosov, M., Kocherlakota, N. R., and Tsyvinski, A. (2003). Indirect and Capital Optimal Taxation. *Review of Economic Studies*, 70(3):569–587.

Judd, K. L. (1985). Redistributive taxation in a simple perfect foresight model. *Journal of public Economics*, 28(1):59–83.

Kaplow, L. (1995). A note on subsidizing gifts. *Journal of Public Economics*, 58(3):469–477.

Kaplow, L. (2001). A Framework for Assessing Estate and Gift Taxation. In W. Gale, J. H. and Slemrod, J., editors, *Rethinking Estate and Gift Taxation*, number January. Brookings Institution Press, Washington, D. C.

Kocherlakota, N. R. (2005). Zero expected wealth taxes: A mirrlees approach to dynamic optimal taxation. *Econometrica*, 73(5):1587–1621.

Kopczuk, W. (2013a). Incentive Effects of Inheritances and Optimal Estate Taxation. *American Economic Review*, 103(3).

Kopczuk, W. (2013b). Taxation of intergenerational transfers and wealth. In *Handbook of public economics*, volume 5, pages 329–390. Elsevier.

Lockwood, B. B. and Weinzierl, M. (2016). Positive and normative judgments implicit in U . S . tax policy , and the costs of unequal growth and recessions $. *Journal of Monetary Economics*, 77:30–47.

McCaffery, E. (1994). The Uneasy Case for Wealth Transfer Taxation. *Yale Law Journal*, 2(104):283–365.

Mirrlees, J., Adam, S., Besley, T., Blundell, R., Bond, S., Chote, R., Gammie, M., Johnson, P., Myles, G., and Poterba, J. (2010). Reforming the tax system for the 21st century: The mirrlees review. *Institute for Fiscal Studies*.

Nordblom, K. and Ohlsson, H. (2006). Tax avoidance and intra-family transfers. *Journal of Public Economics*, 90(8-9):1669–1680.

OCDE (2021). *Inheritance Taxation in OECD Countries*.

Piketty, T. and Saez, E. (2012). A theory of optimal capital taxation. Technical report, National Bureau of Economic Research.

Piketty, T. and Saez, E. (2013). A theory of optimal inheritance taxation. *Econometrica*, 81(5):1851–1886.

Saez, E. and Stantcheva, S. (2016). Generalized social marginal welfare weights for optimal tax theory. *American Economic Review*, 106(1):24–45.

Stantcheva, S. (2015). Optimal income, education, and bequest taxes in an intergenerational model. Technical report, National Bureau of Economic Research.

Straub, L. and Werning, I. (2020). Positive long-run capital taxation: Chamley-judd revisited. *American Economic Review*, 110(1):86–119.

Wilhelm, M. O. (1996). Bequest behavior and the effect of heirs' earnings: Testing the altruistic model of bequests. *The American Economic Review*, pages 874–892.