On the Impact of Telecommuting on Cities*

Toshitaka Gokan† Sergey Kichko‡ Jacques-François Thisse§

October 25, 2021

Abstract

We study the impact of telecommuting in a monocentric city which produces (i) a tradable consumption good using skilled and unskilled labor and (ii) a non-tradable consumer service provided by unskilled workers at the city center to the skilled workers. Commuting costs are proportional to wages. When the WFH share is low, the skilled reside near the CBD and all workers earn more under WFH. By contrast, a high WFH share lowers both wages and leads the skilled to reside in the suburbs. Telecommuting leads to lower urban costs in the latter case, but not in the former. We then consider two cities that have different productivities. WFH allows skilled workers of the more productive city to reside in the less productive city where housing is cheaper while keeping their job in the more productive city. The flow of this type of inter-city commuters first increases and, then, decreases with the WFH share. Likewise, skilled workers of the less productive city may take a job in the more productive city while keeping their residence in the less productive city. The flow of the second type of inter-city commuters increases with the WFH share. For these commuting patterns to arise, the two employment centers must be connected by a link that allows workers to travel at relatively low costs.

Keywords: telecommuting, working from home, gentrified cities, doughnet cities, inter-city commuting

JEL Classification: J60, R00

*We are very grateful to Masahisha Fujita and Giordano Mion for insightflul discussions. This research was undertaken within the framework of the HSE University Basic Research Program and funded by the Russian Academic Excellence Project '5-100'.

†Institute of Developing Economies - JETRO. E-mail: Toshitaka_Gokan@ide.go.jp

‡HSE University. E-mail: skichko@hse.ru

§HSE University, Institute of Developing Economies - JETRO and CEPR. E-mail: jacques.thisse@uclouvain.be

1 Introduction

Ever since the beginning of the Covid-19, there has been a growing, but still limited, interest in the effects of *telecommuting*—also called working from home (WFH) or telework—on the way firms and people organize their activities across space. Telework cuts commuting time and costs, reduces traffic congestion and the emission of greenhouse gases. The relative lack of interest among urban economists for WFH is, therefore, surprising because it has long been recognized that business districts and commuting between workplaces and residences are key elements in the way cities are organized. Since telecommuting may become a permanent feature of our economies, we may expect cities to be subject to a systemic shock whose consequences are still unclear (Barrero *et al.*, 2021). Therefore, the issue is important from the policy viewpoint, as well as for urban economics. The goal of this paper is to study how different intensities of telecommuting affect the spatial and social organization of cities, their productivities and their residents. We first discuss the case of one city and, then, turn our attention to a system of two cities where individuals can reside in one city and work in the other.

To address these questions, we develop a setting which accounts for the major features of WFH. First, despite a few exceptions (think of call-centers), telework characterizes predominantly the skilled workers. Dingel and Neiman (2020) observe that the 37% of US jobs that can be performed at home pay more than those that have to be performed in the workplace. Adams-Prassl *et al.* (2020) find that workers with a university degree can do a significantly higher share of their work tasks from home, while Mattana *et al.* (2020) observe that in Denmark the Covid-19 has a strong negative impact on individuals with low education or vocational training, probably because their jobs are not portable.

Second, after decades of flight to the suburbs, city centers have again become desirable places where to live. According to the 2020 Census, each of the 10 largest U.S. urban centers gained population for the first time ever since 1950 (but they are still outpaced by the suburbs). This trend is partly rooted in the shift toward a knowledge-based economy and is embodied in an expanding class of highly-educated and young professionals. These workers spend a large number of hours at their jobs, which explains their growing distaste for commuting and their growing taste for urban amenities supplied near the workplaces (Moretti, 2012; Edlund *et al.*, 2015; Couture and Handbury, 2020). This has fostered the supply of local consumption services (LCS), which are available in city centers and produced by low-pay workers. Bloom (2020) predicts that WFH could remove up to 50% of spending in city centers. In a detailed study of service jobs provided in UK business districts, De Fraja *et al.* (2021) find that the demand for LCS will decrease by 20% if half of the commuters work home two days per week. Althoff *et al.* (2021) conclude that "low-skill service workers in big cities bore most of the recent pandemic's economic impact".

To achieve our goal, we develop a monocentric city model that takes into account the above features. There are three primary production factors—land, skilled, and unskilled labor—and two sectors—the first sector produces the consumption (costlessly tradable) good using skilled and unskilled labor, while the second sector supplies (non-tradable) LCS using unskilled labor. Home labor and office labor are combined through a general transformation function that captures the idea that the current ICT are such that very low or very high levels of telework are (today) relatively inefficient (Behrens *et al.*, 2021; Davis *et al.*, 2021). This function generates a new effect that becomes concrete through the spatial redeployment of skilled workers. Since commuting is central for our purpose, we account for the well-documented fact that commuting costs increase with incomes (Small, 2012; Koster and Koster, 2015).

Our main findings may be summarized as follows. To start with, we consider the case of a closed city. First, in the absence of telecommuting, the city is *gentrified*, i.e., the skilled set up around the center because they have higher incomes and higher commuting costs than the unskilled. Provided that the WFH share is not too large, the social structure of the city remains the same. Once the WFH share goes beyond a certain threshold, the skilled find it desirable to reside in the periphery where land is cheaper as they seldom commute to the CBD. We refer to this urban structure as the *doughnut city* because the city gets less and less vibrant with a melting consumption of LCS and a growing mass of unskilled who now work in the final sector for a lower pay. In sum, *a wide-spreading of telecommuting is likely to trigger a re-suburbanization of the skilled at the expense of the unskilled workers*, which agrees with the recent trend toward more suburbanization in several US metropolitan areas (Gupta *et al.*, 2021; Liu and Su, 2021).

Second, as expected, a more skilled city makes the gentrified structure more robust to growing WFH. Less expected, a gentrified city where workers consume more LCS is less robust to a strong WFH shock because the wage gap between skilled and unskilled is narrower. Hence, cities with a vibrant downtown are likely to be affected by strong external shocks such as the COVID because office workers stay away while there are few reasons for visitors to spend time or money there on the weekend. Third, telework has various redistributive effects. First of all, regardless of the city's social structure a higher WFH share exacerbates wage disparities between the skilled and the unskilled because unskilled jobs are destroyed in the LCS sector. The unskilled then move to the final sector, which boosts the skilled wage and depresses the unskilled wage. Furthermore, the wages of both types of workers is bell-shaped in the WFH share. The top of the bell for the unskilled occurs for a value of the WFH share which is smaller than that corresponding to the top of the bell for the skilled. In other words, *a low WFH share allows all workers to earn more while a high share lowers the income of both types of workers.*

Last, the general belief is that telecommuting leads to lower commuting costs and housing expenditure, and thus makes the teleworkers better-off. This argument overlooks the impact of telecommuting on the those who have to commute every working day, mostly the unskilled. However, their income and location are affected by telecommuting. Furthermore, because telecommuting allows the teleworkers to earn more over some domain of values of the WFH shares, their commuting costs increase despite the lower frequency of trips. Hence, in a gentrified city, the skilled pay higher urban costs when WFH arises. The same holds for the unskilled as their wage also rises for small values of WFH share. By contrast, in a doughnut city, WFH leads to a lower land rent everywhere in the city. In sum, telecommuting leads to lower urban costs in a doughnut city, but not in a gentrified city. As a result, average housing expenditure across skilled is lower in a doughnut city than in a gentrified city. This highlights one of the main gains that the skilled expect from home-working.

We now turn our attention to an urban system formed by two cities that differ in their total factor productivity. In this context, urban costs are higher in the more productive city. Consequently, some skilled may choose to reside in the less productive city while keeping their job in the more productive city. For this to happen, travelling (or commuting) between the two cities must be cheap enough. This happens when the two cities are connected by a high-speed railway or a highway that allows *between-city commuters* to travel at low cost between the two employment centers. Such a commuting pattern gives rise to a new kind of suburbanization in which the size of the more productive city shrinks while the size of the less productive city grows. However, between-city commuting leads to lower urban costs in the more productive city but raises them in the less productive one. Therefore, the departure of inhabitants—but not of workers— from the more productive city makes those who stay better-off, whereas the arrival of new residents in the less productive city—who do not work therein—makes the incumbents worse-off. This shows how inter-city commuting differs from inter-city migration studied in standard models of urban systems.

Our main result is that *the mass of between-city commuters is bell-shaped in the WFH share.* For small values of the WFH share, the skilled who reside in the more productive city still commute quite frequently, and thus bear relatively high commuting costs. As WFH grows, more skilled find it optimal to move to the less productive city and to set up in the vicinity of its center, pushing the skilled who work there toward an intermediate area situated between the between-city commuters and the local low-skilled workers. When the WFH share rises further, the difference between urban costs shrinks, which leads between-city commuters to save the cost of travelling between cities by moving back to their original city. Simultaneously, those who remain between-city commuters will locate in the intermediate area of the less productive city where housing is less expensive. The central areas of both cities are now inhabited by the unskilled because the skilled need to commute less and, therefore, prefer to live in the suburbs where land is cheap. Last, when the WFH share is sufficiently high, all between-city commuters may be back to their city of origin because the land rent is low while commuting is almost costless due to its low frequency. In this case, the more productive city regains residents but LCS establishments patronized by commuters suffer even more because the WFH share has further decreased.

Inter-city commuters have their counterpart, i.e., *between-city workers.* When a city is more productive than another, the skilled who live in the latter may want to take a job in the former where wages are higher while keeping their residence in their city of origin. Again, for this to arise, travelling between the two city centers must be (relatively) inexpensive. Such a commuting leads to the expansion of the output of the more productive city and to the reverse in the other city. Furthermore, the arrival of between-city workers triggers a revival of the city center at the expense of that of their original workplace. As a result, fewer unskilled now work in the final sector of the more productive city. Put together, these two effects increase the mass of skilled and decrease the mass of unskilled working in the final sector of the more productive city. This in turn leads to higher wages for the unskilled and lower wages for the skilled. Simultaneously, the flow of between-city workers toward the more productive city exacerbates wage inequality in the less productive city because the skilled ratio falls. Therefore, *the presence of between-city workers shrinks the wage gap in the more productive city but exacerbates economic inequality in the less productive city.*

In the whole paper, we assume that the WFH share is exogenous. Of course, we expect this share to be chosen endogenously through agents' decisions. However, it is still unclear how this share will be determined, e.g., by firms or by workers or though negotiations between employers and employees. The adopted rules are likely to vary across sectors and countries, and may even differ between firms. In any case, whatever the institutional mechanism used, a certain value of the WFH share will emerge and will have consequences for cities similar to those discussed in this paper.

Related literature. While the management and psychological literature on telecommuting is mounting—the survey by Allen *et al.* (2015) cites about 200 papers—the economic theory literature on telecommuting and cities is meager. We have found only two theoretical papers published in the *Journal of Urban Economics*. Safirova

(2002) extends the monocentric city model to account for telecommuting when home workers and office workers are imperfect substitutes. Safirova remains within standard urban economics by considering a land market and a single sector producing the consumption good. Moreover, she provides only numerical solutions. Rhee (2008) studies the trade-off between working time and leisure and shows that most of the commute time saved by WFH is allocated to work rather than leisure. Recent papers, such as Althoff *et al.* (2020), Behrens *et al.* (2021), De Fraja *et al.* (2021), Delventhal *et al.* (2021), and Koren and Peto (2020), study different relationships between telecommuting and the spatial organization of activities. Since our aim is to propose an integrated setting, we borrow at will from these contributions and compare results in due course. Brueckner *et al.* (2021) explore the case of between-city commuting. However, they assume that commuting costs that do not vary with teleworkers' income. This assumption appears to be crucial as our analysis shows that telecommuting affects the income of both skilled and unskilled workers in a non-trivial way, which in turn alters commuting costs and land rents.

The remainder of the paper is organized as follows. The model is presented in Section 2. In Section 3, we determine the skilled and unskilled wages in a closed city. Section 4 characterizes the equilibrium of the city structure for different values of the WFH share. In Section 5, we study the patterns of commuting between cities which arise together with WFH. To keep things tractable, we consider two cities that have different productivities. Section 6 and 7 focus respectively on between-city commuters and between-city workers. Section 8 concludes by discussing some extensions.

2 The model

We consider a one-dimensional space $X = [0, \infty)$ with a dimensionless central business district (CBD) located at $0 \in X$. We denote by $x \in X$ a location and its distance to the CBD. The land density at each location is one and the opportunity cost of land is zero. There are three primary production factors: land, skilled labor (s) and unskilled labor (ℓ). The mass of k-workers is given by L_k for $k = \ell, s$ with $L_\ell > L_s$. Each k-worker supplies inelastically one unit of her type of labor. Both types of workers consume one unit of housing (land) and work at the CBD. The economy is formed by two sectors, the first one produces a costlessly tradable consumption good (c), which is chosen as the numéraire, and the second sector supplies local consumption services (LCS) which are non-tradable and non-teleworkable. In line with urban economics, we assume absentee landowners.

2.1 Consumption

The skilled may split their working time between home, $\rho \in [0, 1]$, and office, $1 - \rho$. In contrast, the unskilled activities are undertaken at the CBD. Commuting costs are linear in the distance x to the CBD. As commuting costs are strongly correlated to labor incomes, we assume that a worker's commuting rate is equal to a given share $\xi > 0$ of her wage, i.e., her commuting rate is equal to $\xi w_k x$ for $k = \ell, s$. We also make the well-established fixed-lot size assumption, that is, each worker uses 1 unit of land for housing and pays the land $R(x)$ when she resides at $x \in X$. Hence, the city size is given by $L_s + L_\ell$.

The skilled also consume η units of LCS provided at the CBD, such as restaurants, bars and other leisure amenities, as well as other personal services consumed near the workplace, which we consider as an index of cultural and social vibrance associated with the gentrification of city centers (Couture and Handbury, 2020). In what follows, we assume that the consumption of LCS is proportional to the time spent working at the CBD, $(1-\rho)\eta$, which decreases when the share of teleworking rises (De Fraja *et al.*, 2021). The supply of urban amenities is thus endogenous through the number of skilled working at the CBD. Although it is reasonable to expect some establishments providing LCS to be drifted from the CBD to the suburbs, we believe that a substantial share of such services will no longer be provided. Indeed, as many consumers are located in sparsely populated areas and have to travel to the establishments that provide LCS, these establishments will have access to a significantly smaller pool of customers than when they are located in the CBD. This in turn should deter entry. We therefore assume that LCS are supplied and consumed at the CBD only. As a result, *a higher WFH share leads the skilled workers to substitute the consumption good for the LCS*. For simplicity, we assume that the individual demand η for LCS is perfectly inelastic because this consumption meets some basic needs like food and socializing. Recall, however, that the total supply of LCS varies with the share of office workers.

By implication of the fixed lot size assumption, a worker's program may be written as follows:

$$\begin{aligned} \max U &= c_s, \qquad \text{s.t. } w_s = R(x) + c_s + p\eta(1-\rho) + (1-\rho)\xi w_s x, \\ \max U &= c_\ell, \qquad \text{s.t. } w_\ell = R(x) + c_\ell + \xi w_\ell x \end{aligned}$$

where $(1-\rho)\eta$ is the consumption of LCS, w_k is the wage paid to a k-worker ($k = \ell, s$), $\xi w_k x$ her commuting cost when she commutes everyday, c_k her consumption of the final good, $R(x)$ the land rent at x, and p the price of the LCS.

On the one hand, each skilled commute less under a higher ρ. On the other, when ρ become sufficiently large, skilled may chose to reside at the periphery, and so they make fewer but longer trips. Therefore, the average commuting costs may increase or decrease with ρ.

2.2 Production

Consumer local services. LCS are produced under constant returns and perfect competition, using unskilled labor. We choose the unit of unskilled labor for a one unskilled worker to produce one unit of LCS, so that $p = w_\ell$. Since the unskilled are perfectly mobile between the LCS and final sectors, they are paid the same wage w_ℓ in the two sectors.

Therefore, for any given ρ, LCS market balance implies that $\eta(1-\rho)L_s$ unskilled workers are employed in the LCS sector while

$$\lambda_c = L_\ell - \eta(1-\rho)L_s \tag{1}$$

unskilled are hired by the final sector.

Consumption good. The final sector is located at the CBD. It operates under constant returns and perfect competition using skilled and unskilled labor. The most controversial issue is probably about the impact of WFH on workers' productivity. Following the pioneering work of Bloom *et al.* (2015) who undertake a field experiment in a Chinese company, the economics literature leans towards the assumption of positive productivity effects because WFH would allow workers to better organize their various business and home tasks (Mas and Pallais, 2020). But is this argument really compelling? Using a Japanese survey that provides the percentage of an employee's productivity under WFH conditions relative to the same employee's productivity at the usual workplace, Morikawa (2020) finds that the productivity during June 2020 was only about 60 to 70% of what it is at the workplace in June 2019. However, the productivity drop is significantly lower for skilled workers and long-distance commuters. On the other hand, Barrero *et al.* (2021) report several experiments where labor flexibility raises productivity and find that 20% of workdays will be provided from home when the pandemic will end. As the string of contributions in management and psychology that come up with diverging results is almost endless, we find it fair to say that there is no consensus in the literature on the productivity effects of WFH. This is probably because the effects of WFH depend on the the individual characteristics of teleworkers, as well as on the specificities of occupations and industries (Adams-Prassl *et al.,* 2020; Bartik *et al.*, 2020).[1] This absence of clear-cut conclusions leads us to remain agnostic about the productivity effects of WFH.

The parameter $\phi \in [0,1]$ characterizes the level of development of ICT that allow individuals to work from home instead of commuting to and working at the firm: more efficient ICT mean a higher ϕ. Note that the presence of a productive advantage associated with better infrastructure and agglomeration economies at the CBD reduces the value of ϕ. In a nutshell, ϕ measures the efficiency of ICT relative to the efficiency of face-to-face communication within firms. The results of Battiston *et al.* (2021) suggest that the former is less efficient than the latter. Likewise, in a study on the spatial organization of MIT, Claudel *et al.* (2017, p.2) demonstrate "the significant role that spatial proximity still plays in collaborative knowledge creation process *despite the abundance of tools for digital communication and virtual collaboration*" (our emphasis). It is, therefore, not too surprising that Davis *et al.* (2021) find an estimated value of ϕ which is much smaller than 1.

We assume that combining $L_h = \phi\rho L_s$ units of home labor and $L_o = (1-\rho)L_s$ units of office labor translates into $f(L_h, L_o) > 0$ efficiency units of skilled labor, where f is homogeneous linear with $f_1 > 0$ $f_{11} < 0$, $f_2 > 0$, and $f_{22} < 0$. These conditions imply $f_{12} > 0$. In other words, home labor and office labor are *complements* in the sense of Milgrom and Roberts (1990), i.e., the marginal productivity of an input rises when the quantity of the other input increases. This agrees with Gaspar and Glaeser (1998) who conclude that face-to-face contacts and ICT are complements rather than substitutes (see also Goldfarb and Tucker, 2019). Furthermore, more efficient ICT (resp., better commuting facilities) increase the productivity of home (resp., office) workers. As a result, we have

$$f(L_h, L_o) = f(\phi\rho L_s, (1-\rho)L_s) = f(\phi\rho, 1-\rho)L_s \equiv A(\rho)L_s.$$

Moreover, no input is essential, that is, $f(L_h, 0) > 0$ and $f(0, L_o) > 0$.

Differentiating twice $A(\rho)$ and using the properties of f shows that $A(\rho)$ is strictly concave (we do not need f to be jointly concave for this).[2] Thus, $A(\rho)$ either increases, or decreases or is single-peaked over $[0,1]$. Therefore, the unique maximizer ρ_M over $[0,1]$ is given by 0, or 1 or belongs to $(0,1)$. Given the current level of development of the information and communication technologies, we find it reasonable to assume that $A(\rho)$ increase at $\rho = 0$. It

[1] Individual productivity also varies subtly with team composition and experience. For example, it is challenging to integrate new workers into existing teams as the former do not have access to the same common tacit-knowledge-pool as the incumbents.

[2] This property is consistent with the "curvilinear" relationship between telecommuting and job satisfaction studied in management as it seems reasonable to expect the worker's output to be highly correlated to her job satisfaction (see Allen *et al.*, 2015, for a detailed survey).

also seems reasonable to assume that the on-going ICT are not sufficiently efficient for $A(\rho)$ to be increasing over the whole interval $[0,1]$. In other words, $A(\rho)$ decreases at $\rho = 1$. When these two conditions hold, $A(\rho)$ is positive and single-peaked over the interval $[0,1]$ with a maximizer at $\rho_A \in (0,1)$. Intuitively, this amounts to assuming that, *for the current stage of development of ICT, a very large or a very small mass of teleworkers is inefficient.*

For a given WFH share $\rho \in [0,1]$, the production function of the final sector is given by

$$F(L_s, \lambda_c) = (A(\rho)L_s)^{\beta}\, \lambda_c^{1-\beta}. \tag{2}$$

Hence, $(A(\rho))^{\beta}$ plays the role of the total factor productivity (TFP) of the final sector, which is single-peaked and maximized at ρ_A.[3]

The WFH share ρ affects the productivity of the final sector via two different channels. First, the WFH share affects the TFP of the final sector through $A(\rho)$. Second, the final sector produces more as a growing mass of unskilled leave the LCS sector and become available for working in the final sector ($\lambda_c \uparrow$), which fosters an output expansion through a higher mass of unskilled workers. This effect arises when ρ increases because the demand for LCS shrinks.

Substituting λ_c given by (1) in (2) yields

$$F(L_s, L_\ell) = (A(\rho)L_s)^{\beta}\, (L_\ell - \eta(1-\rho)L_s)^{1-\beta},$$

which increases with L_s for all ρ if and only if

$$\boldsymbol{L} > \frac{\eta}{\beta}, \tag{3}$$

where $\boldsymbol{L} \equiv L_\ell / L_s$ is the city's skill ratio. In what follows, we assume that (3) holds for the production function to increase with the quantity of each production factor. It then follows from (3) that λ_c given by (1) is positive because $\beta < 1$.

3 The equilibrium conditions and preliminary results

3.1 Labor market

Using $\beta F = w_s L_s$ and $(1-\beta)F = w_\ell \lambda_\ell$, we obtain the following system of two equations for the skilled and unskilled wages:

$$w_s = \beta\, (A(\rho)L_s)^{\beta} \left(\frac{\lambda_c}{L_s}\right)^{1-\beta}, \qquad w_\ell = (1-\beta)\, (A(\rho)L_s)^{\beta} \left(\frac{L_s}{\lambda_c}\right)^{\beta}. \tag{4}$$

Hence, home-working has an impact on the productivity of unskilled through the productivity of the final sector.

Plugging λ_c into (4) yields the skilled and unskilled wages for any given ρ:

$$w_s = \beta\, (A(\rho)L_s)^{\beta}\, (\boldsymbol{L} - \eta(1-\rho))^{1-\beta}, \qquad w_\ell = \frac{1-\beta}{\beta} \frac{1}{\boldsymbol{L} - \eta(1-\rho)} w_s. \tag{5}$$

Hence, the skilled wage increases when the share of unskilled rises while the unskilled wage decreases. Furthermore, a higher consumption of LCS reduces the skilled wage but raises the unskilled wage because the unskilled face a broader range of job opportunities.

[3] Our analysis can be extended to account for two well-documented facts, i.e., the presence of knowledge spillovers among the skilled and a human capital externality from the skilled to the unskilled. This can be done by assuming that the TFP is given by $((1-\rho)L_s)^{\gamma}\, (A(\rho))^{\beta}\, L_s^{\beta}$ where $0 < \gamma < 1$ (Combes and Gobillon, 2015). This amounts to assuming that the skilled share is higher. However, WFH weakens the TFP of the final sector through $(1-\rho)^{\gamma}$. In this case, WFH may me harmful to the economy through foregone spatial externalities.

The wage ratio is given by

$$\frac{w_s}{w_\ell} = \frac{\beta}{1-\beta}\left(\boldsymbol{L} - \eta(1-\rho)\right), \tag{6}$$

which increases with ρ because the concomitant drop in the consumption of LCS makes the final sector relatively more unskilled labor intensive. In other words, *income inequality between skilled and unskilled workers widens as the WFH share increases.*

Empirical evidence suggests that there is skilled wage premium in cities, that is, the skilled wage is higher than the unskilled wage for all ρ. It then follows from (6) that

$$\boldsymbol{L} > \frac{1-\beta}{\beta} + \eta, \tag{7}$$

must hold, which is a more stringent condition than (3) if and only if $\eta > 1$.

3.2 Wages

The impact of the WFH share on wages is less straightforward because they both depend on $A(\rho)$ whose behavior is not monotone. To provide a clear-cut discussion, consider the example where $A(\rho)$ is given by a CES function:

$$A(\rho) = \left[\phi\rho^{\frac{\sigma-1}{\sigma}} + (1-\rho)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}, \tag{8}$$

where $\sigma > 1$ is the elasticity of substitution between home and office workers. Clearly, (8) is concave and both $A(0)$ and $A(1)$ are positive. According to Davis *et al.* (2021), σ is approximately equal to 5. In this section, we will use (8) for $A(\rho)$ but we obtain the other results for a nonspecific concave function $A(\rho)$.

Plugging (8) in (5) and differentiating the corresponding exression with respect to ρ yields:

$$\frac{\mathrm{d}w_\ell}{\mathrm{d}\rho} = \frac{1-\beta}{[\boldsymbol{L} - \eta(1-\rho)]^2}\,\frac{\phi(\boldsymbol{L}-\eta)\rho^{-\frac{1}{\sigma}} - \boldsymbol{L}\,(1-\rho)^{-\frac{1}{\sigma}}}{\phi\rho^{\frac{\sigma-1}{\sigma}} + (1-\rho)^{\frac{\sigma-1}{\sigma}}}w_s. \tag{9}$$

Hence, the unskilled wage increases for $\rho < \rho_\ell$ and decreases over $(\rho_\ell, 1]$ where

$$\rho_\ell \equiv \frac{\phi^\sigma}{\left(\frac{\boldsymbol{L}}{\boldsymbol{L}-\eta}\right)^\sigma + \phi^\sigma} < \rho_A.$$

This shows that the interval over which the unskilled wage increases with the WFH share becomes wider as more efficient ICT are developed, the share of unskilled workers gets higher, and the consumption of LCS is lower.

As for the skilled wage, taking the derivative of w_s results in

$$\frac{\mathrm{d}w_s}{\mathrm{d}\rho} = \frac{1}{\boldsymbol{L} - \eta(1-\rho)}\left[\beta\frac{\phi(\boldsymbol{L}-\eta)\rho^{-\frac{1}{\sigma}} - \boldsymbol{L}\,(1-\rho)^{-\frac{1}{\sigma}}}{\phi\rho^{\frac{\sigma-1}{\sigma}} + (1-\rho)^{\frac{\sigma-1}{\sigma}}} + \eta\right]w_s. \tag{10}$$

The first term between brackets is positive for $\rho < \rho_\ell$ and negative otherwise. Since this term tends to $-\infty$ as $\rho \to 1$ while η is positive, there exists a unique value ρ_s such that the skilled wage increases for $\rho < \rho_s$ and decreases for $\rho > \rho_s$. In addition, we have $\rho_s > \rho_\ell$ because (10) is positive at ρ_ℓ. Both the unskilled and skilled wages are thus bell-shaped over $(0, 1)$, and the unskilled wage starts decreasing with the WFH share before the skilled wage. This generalizes Behrens *et al.* (2021) who consider a multi-sectoral setting with one business district and one residential area. Moreover, since ρ_A increases with ϕ, more efficient ICT ($\phi \uparrow$) allow both types of workers to benefit from rising wages over wider intervals of WFH shares ($\rho_\ell \uparrow$ and $\rho_s \uparrow$), while $w_s(\rho_A)$ and $w_\ell(\rho_A)$ also rise with ϕ. Therefore, more efficient ICT lean toward higher wages for all workers.

Using (4), the output of the final sector (2) takes the form

$$F = \frac{L_s^{1-\beta}}{\beta} w_s, \tag{11}$$

so that the skilled wage and the output of the final sector both increase with the WFH share over $[0, \rho_s)$ and, then, decrease when ρ is larger than ρ_s.

It remains to determine the impact of ρ on the total income, which is given by $L_s w_s + L_\ell w_\ell$. Using (9) and (10), it is readily verified that the derivative of the total income is such that

$$\frac{\mathrm{d}}{\mathrm{d}\rho}(w_s L_s + w_\ell L_\ell) = \frac{w_s L_s}{\boldsymbol{L} - \eta(1-\rho)} \left[\frac{\boldsymbol{L} - \beta\eta(1-\rho)}{\boldsymbol{L} - \eta(1-\rho)} \frac{\phi(\boldsymbol{L} - \eta)\rho^{-\frac{1}{\sigma}} - \boldsymbol{L}\,(1-\rho)^{-\frac{1}{\sigma}}}{\phi\rho^{\frac{\sigma-1}{\sigma}} + (1-\rho)^{\frac{\sigma-1}{\sigma}}} + \eta \right].$$

The first term between brackets is a product of positive term and a term which is positive for $\rho < \rho_\ell$ and negative for $\rho > \rho_\ell$ while the second positive term η is a constant that serves as a shifter. Furthermore, the bracketed term is negative at $\rho = \rho_s$. Therefore, the total income increases for $\rho \leq \rho_\ell$, decreases for $\rho \geq \rho_s$, and reaches its peak in (ρ_ℓ, ρ_s). We will see that the output of the final sector may increase or decrease when the city shifts from a gentrified structure to a doughnut because this change arises for a value of ρ that typically belongs to that interval.

Summarizing yields the following proposition.

Proposition 1. *For a given city structure, the skilled and unskilled wages, as well as the total income, are bell-shaped in the WFH share. Furthermore, a higher WFH share exacerbates wage inequality between the skilled and the unskilled.*

In other words, telecommuting has implications for economic inequality on top of it effects on wages, an aspect that is often absent from the debate about its costs and benefits. In particular, Proposition 1 implies the gap between commuting rates widens when the WFH share rises.

3.3 Commuting costs

As the unskilled commuting cost $\xi w_\ell x$ is proportional to wage, Proposition 1 implies that an unskilled worker located at any given x bears a commuting cost that first increases and, then, decreases with the WFH share. As for the skilled workers, their commuting cost $(1-\rho)\xi w_s x$ varies with ρ as $(1-\rho)w_s(\rho)$. Using (10), we have

$$\frac{\mathrm{d}}{\mathrm{d}\rho}((1-\rho)w_s(\rho)) = \left\{ \frac{1-\rho}{\boldsymbol{L} - \eta(1-\rho)} \left[\beta \frac{\phi(\boldsymbol{L} - \eta)\rho^{-\frac{1}{\sigma}} - \boldsymbol{L}\,(1-\rho)^{-\frac{1}{\sigma}}}{\phi\rho^{\frac{\sigma-1}{\sigma}} + (1-\rho)^{\frac{\sigma-1}{\sigma}}} + \eta \right] - 1 \right\} w_s. \tag{12}$$

Clearly, this derivative is positive for small values of ρ because the term in squared brackets tends to ∞ when $\rho \to 0$. When ρ reaches ρ_ℓ, (12) is negative when $\eta < (1-\beta)/\beta$. More generally, (12) is negative for $\rho \geq \rho_s$ because the term in squared brackets is zero for $\rho = \rho_s$ and negative for $\rho > \rho_s$. As a result, (12) reaches its peak when $\rho < \rho_s$. Then, we may conclude as follows.

Proposition 2. *At any city location, the commuting cost paid by a skilled or an unskilled worker increases when the WFH share is low while it decreases when this share is high.*

This proposition runs against the general belief according to which commuting costs should decrease with telecommuting. However, this argument disregards the fact that commuting costs vary with workers' income level, which increases when WFH starts rising. Even the unskilled who do not work home may have to spend more on commuting when ρ is small because their income increases.

In the foregoing, we have assumed that a worker keeps living at the same location x regardless of the value of ρ. This is likely to be so for a marginal change in ρ. By contrast, we will see in the next section that a worker may want to change her residence when the WFH share substantially rises.

4 The city structure

The bid rent of a skilled worker at $x \in X$ is defined by $\Psi_s(x) = w_s - c_s - \eta(1-\rho)w_\ell - (1-\rho)\xi w_s x$, while the bid rent function of an unskilled is given by $\Psi_\ell(x) = w_\ell - c_\ell - \xi w_\ell x$. Comparing the slopes of two bid rent functions shows that the skilled outbid the unskilled near the CBD and locate in the area $(0, L_s)$ if

$$\left|\frac{\partial \Psi_s(x, U_s)}{\partial x}\right| > \left|\frac{\partial \Psi_\ell(x, U_\ell)}{\partial x}\right| \Leftrightarrow \frac{w_s}{w_\ell} > \frac{1}{1-\rho} \geq 1, \tag{13}$$

where the equality holds if and only if $\rho = 0$. On the other hand, the unskilled set up near the CBD in the area $(0, L_\ell)$ if $w_s/w_\ell < 1/(1-\rho)$. Thus, two city structures may emerge (Fujita, 1989). Since $1/(1-\rho)$ increase with ρ, a higher WFH share may shift the economy from the former configuration to the latter.

From now on, we say that a city is *gentrified* when the skilled workers choose to set up in the vicinity of the CBD. Otherwise, we have a *doughnut* city in which the city's central area is occupied by the unskilled.

Plugging (6) into (13) shows that the city is gentrified if and only if

$$\frac{\beta}{1-\beta}[\boldsymbol{L} - \eta(1-\rho)] > \frac{1}{1-\rho}, \tag{14}$$

which amounts to the condition

$$\eta\rho^2 + (\boldsymbol{L} - 2\eta)\rho - \left(\boldsymbol{L} - \eta - \frac{1-\beta}{\beta}\right) < 0. \tag{15}$$

The left-hand side of this inequality has only one positive zero because the parabola is convex and has a negative intercept due to (7). Since (15) does not hold when $\rho = 1$, the solution $\bar{\rho}$ to (15) must be smaller than 1. Consequently, *the city is gentrified if and only if the WFH share is not too high*, that is, $\rho \in [0, \bar{\rho})$. Note that the results obtained in Section 3 are valid for both configurations because wages are not affected by the city structure.

Solving (15) for ρ, we obtain

$$\bar{\rho} = 1 - \frac{\boldsymbol{L} - \sqrt{\boldsymbol{L}^2 - 4\eta\frac{1-\beta}{\beta}}}{2\eta}. \tag{16}$$

By conducting a numerical analysis for plausible empirical values for the model parameters, we can gain insights about the relationship between $\bar{\rho}$ and the thresholds $\rho_s > \rho_\ell$. Do the skilled/unskilled wages start decreasing with ρ before or after the city switches from a gentrified to a doughnut center? We assume $\beta = 0.4$, which means a skilled share equal to 40% in the final sector. Since we have no idea about the value of η, we choose $\eta = 1$. For $\boldsymbol{L} = 3$, which corresponds to a share of college educated workers equal to 25% of the population, we obtain $\bar{\rho} = 0.37$. Since ρ_ℓ increases with ϕ and $\phi < 1$, by setting $\phi = 1$ we find the upper bound for ρ_ℓ, which is equal to $0.12 < \bar{\rho}$. For a larger share of unskilled, $\boldsymbol{L} = 4$, we obtain $\bar{\rho} = 0.58$ while the upper bound of ρ_ℓ is 0.19, which is again smaller than $\bar{\rho}$. This same holds true for larger $\boldsymbol{L}$. From now on, we therefore assume that $\rho_\ell < \bar{\rho}$. In other words, the unskilled wage starts decreasing with the WFH share when the city is gentrified.

Since $\bar{\rho}$ is independent of ϕ while ρ_s increases with ϕ, there exists a threshold ϕ^* such that $\rho_s < \bar{\rho}$ if and only if $\phi < \phi^*$. For $\boldsymbol{L} = 3$, we get $\phi^* = 0.44$, while $\boldsymbol{L} = 4$ yields $\phi^* = 0.71$. We may then conclude as follows: when the city is gentrified, raising the WFH share first increases and, then, decreases the skilled and unskilled wages when ϕ is not too large ($\phi < \phi^*$). By contrast, if $\phi > \phi^*$, in a gentrified city the skilled wage always increases while the unskilled wage first increases and, then decreases with ρ. In this case, the skilled wage keeps rising until ρ_s even when the periphery is inhabited by the skilled ($\rho > \bar{\rho}$).

Finally, that combining telecommuting and a variable land consumption should foster a more rapid suburbanization of the skilled, that is, for a WFH share smaller than $\bar{\rho}$, because their higher income allows them to consume more land than the unskilled.

4.1 The gentrified city

Skill and LCS. Raising β shifts the parabola (15) downward, which means that $\bar{\rho}$ increases. Indeed, as shown by (6), a higher β widens the wage gap, which makes it easier for the skilled to outbid the unskilled. In contrast, $\bar{\rho}$ decreases with $\boldsymbol{L}$ because the wage gap shrinks. Since both $1/\boldsymbol{L}$ and β can be used to assess the degree of skillfulness of a city, they have the same impact on the city structure: a more skilled city raises $\bar{\rho}$ and makes the gentrified city more robust to changes in WFH.

More surprising, differentiating $\bar{\rho}$ with respect to η shows that $\bar{\rho}$ decreases with η. Put differently, cities with a more vibrant night life (higher η) is less sustainable to WFH shock. The intuition for this result is as follows. As higher η means that more unskilled are employed in the LCS sector, unskilled labor is a relatively scarce resource in the consumption sector. This drives up the unskilled wage. As skilled labor is relatively abundant, its price diminishes with η. Therefore, for any WFH share, the wage gap between the two groups is narrower for higher η, as captured by (6). Since commuting costs are proportional to wages, the lower wage gap allows the unskilled to outbid the skilled in the central area for lower value of the WFH share.

To sum up,

Proposition 3. *If $\rho < \bar{\rho}$, then the city is gentrified. Furthermore, a gentrified city with a higher share of skilled is more robust to an expansion in home-working while a gentrified city with a higher consumption of LCS is less robust.*

Housing and land rent. When city is gentrified, the worker at the city limit is an unskilled who bears the urban cost $\xi w_\ell(L_s + L_\ell)$ because the opportunity cost of land is zero. Since all the unskilled reach the same utility level at the spatial equilibrium, they share the urban costs which is given by

$$UC_\ell = \xi w_\ell(L_s + L_\ell). \tag{17}$$

Furthermore, the equilibrium land rent over $(L_s, L_s + L_\ell]$ is given by

$$R_\ell(x) = \xi w_\ell(L_s + L_\ell - x), \tag{18}$$

which is linear and downward-slopping in x over $(L_s, L_s + L_\ell]$. In addition, the land gradient at $x \in (L_s, L_s + L_\ell]$ increases with the WFH share when $\rho < \rho_\ell$ and, then, decreases until $\bar{\rho}$. In a gentrified city, *telecommuting implies that the unskilled spend more on housing when* $\rho < \rho_\ell$ because they earn more and bear higher commuting costs.

As $R_\ell(L_s) = R_s(L_s)$, the spatial equilibrium condition for the skilled implies that the urban costs at each location $x \in [0, L_s)$ are equal and such that

$$UC_s(x) = R_s(x) + (1-\rho)\xi w_s x = \xi w_\ell L_\ell + (1-\rho)\xi w_s L_s \equiv UC_s. \tag{19}$$

Equations (9) and (12) show that both terms of UC_s in (19) increase for small values of WFH share. This has the following unexpected consequence: *in a gentrified city, the skilled bear higher urban costs when WFH arises because their income, hence their commuting cost, increase.* However, urban costs decrease with ρ when $\rho \geq \rho_s$ because both terms in (19) decrease. In this case, both the skilled wage and urban costs decrease with the WFH share. Furthermore, UC_s decrease for $\rho \geq \rho_\ell$ when $(1-\beta)/\beta > \eta$. In this case, for $\rho_\ell < \rho < \rho_s$, the skilled wage increases while their urban costs decrease.

Regarding the equilibrium land rent over $[0, L_s)$, it is given by

$$R_s(x) = \xi w_\ell L_\ell + (1-\rho)\xi w_s (L_s - x) > 0, \tag{20}$$

which is linear and downward-slopping in x.[4] Equations (19) and (20) imply that the skilled land rent exhibits the same pattern as the skilled urban costs. In particular, *the land rent in the central area, hence in the whole city, increases when WFH arises.*

In a gentrified city, the skilled land gradient is equal to $-(1-\rho)\xi w_s$ while the unskilled land gradient is $-\xi w_\ell$. Since $(1-\rho)w_s > w_\ell$, the land rent is steeper over $[0, L_s)$ than over $(L_s, L_s + L_\ell]$. This is because the WFH share is still low enough for the skilled commuting rate to be higher than the unskilled commuting rate. Hence, the land rent has an outward kink at $x = L_s$. Furthermore, according to the value of ρ, a higher WFH share may increase or decrease the land gradient, thus making the land rent at a location x steeper or flatter.

Finally, we use (20) to compute the average housing expenditure made by the skilled in a gentrified city, which is equal to $R_s = \xi w_\ell L_\ell + (1-\rho)\xi w_s L_s/2$. As we are unable to find the peak of $(1-\rho)w_s$, we numerically plot R_s for different empirically plausible values of the parameters. They all show that the land rent $R_s(x)$ is shifted downward as ρ increases for positive but low values of ρ. More specifically, we find that R_s reaches its highest value for $\rho < 0.04$.

As for the unskilled, their average land rent is equal to $\xi w_\ell L_\ell/2$, which is bell-shaped with a peak at ρ_ℓ. As a result, *when the city is gentrified the unskilled spend more on housing when the WFH share is low, whereas the skilled spend less unless the WFH share is very small.* It is noteworthy that a higher/lower housing expenditure is not caused here by a larger/smaller land consumption or by a social reorganization of the city. It is due to the interplay between the WFH share, wages and commuting rates. This is to be contrasted with the case of exogenous commuting rate because the land rent always becomes flatter when ρ rises. The same holds when wages are not (yet) adjusted to a substantial hike in the WFH share, like during the COVID lockdowns (Gupta *et al.*, 2021).

The next proposition provides a summary.

Proposition 4. *In a gentrified city, both the skilled and unskilled workers spend more on housing when WFH*

[4] Observe that the land rents satisfy the condition $R_\ell(x) = R_s(x)$ for all $x \in [0, L_\ell + L_s]$ when $\rho = \bar{\rho}$ because $w_\ell(\bar{\rho}) = (1-\bar{\rho})w_s(\bar{\rho})$.

arises. Beyond some threshold, the housing expenditure of the skilled decreases and, then, that of the unskilled also decreases.

4.2 The doughnut city

If the WFH share is sufficiently large ($\rho > \bar{\rho}$), there is a reversal in the city's social structure: the skilled choose residences in the city periphery while the unskilled live in the central area $[0, L_\ell)$. Due to home working, the proximity to the CBD ceases to be a concern to the skilled who choose instead to consume more of the final good by residing in the periphery where housing is cheaper.

Since the opportunity cost of land is zero, we have $R_s(L_s + L_\ell) = 0$ so that the skilled bear urban costs equal to

$$UC_s(x) = R_s(x) + (1-\rho)\xi w_s x = (1-\rho)\xi w_s(L_s + L_\ell) \equiv UC_s,$$

which decreases over $(\bar{\rho}, 1]$ for our preferable set of parameter values. The same does the land rent at $x \in (L_\ell, L_\ell + L_s]$ which is given by

$$R_s(x) = (1-\rho)\xi w_s(L_s + L_\ell - x). \tag{21}$$

The land rent at the boundary between the skilled and unskilled areas is given by $R_s(L_\ell) = (1-\rho)\xi w_s L_s$. Therefore, the urban costs of the unskilled located in the central area are given by

$$UC_\ell(x) = R_\ell(x) + \xi w_\ell x = \xi w_\ell L_\ell + (1-\rho)\xi w_s L_s = UC_\ell(L_\ell),$$

which decreases with ρ in a doughnut city because both terms are decreasing functions in ρ.

The land rent over the central area $[0, L_\ell)$ is given by

$$R_\ell(x) = (1-\rho)\xi w_s L_s + \xi w_\ell \left(L_\ell - x\right), \tag{22}$$

which decreases with the distance to be CBD. In addition, the land rent at any location of the area occupied by the unskilled is shifted downward when the WFH share increases, very much like the land rent over the area occupied by the skilled. By contrast, we have seen that the land rent is shifted upward in a gentrified city when ρ is small In other words, the land market reacts differently to a hike in the WFH share in a gentrified or doughnut city.

It follows from (20) and (21) that, in a gentrified city, the average skilled housing expenditure is equal to $(1-\rho)\xi w_s L_s/2 + \xi w_\ell L_\ell$ whereas this expenditure is given by $(1-\rho)\xi w_s L_s/2$ in a doughnut city. Since $\xi w_\ell L_\ell$ is positive while $(1-\rho)\xi w_s L_s/2$ decreases with ρ for $\rho > \bar{\rho}$, we may conclude that on average *the skilled spend less on housing in a doughnut city than in a gentrified city.* In other words, the housing benefits associated with telecommuting always arise once the city has a doughnut structure, but this need not be so when the city is gentrified because commuting costs are too high.

Computing the average land rent paid by the unskilled in a doughnut city yields the expression $(1-\rho)\xi w_s L_s + \xi w_\ell L_\ell/2$, while the average land rent in a gentrified city is equal to $\xi w_\ell L_\ell/2$. The comparison between the two expressions in not clear. Indeed, even though the unskilled wage is lower in a doughnut city, the average land rent involves the additional positive term $(1-\rho)\xi w_s L_s$. We thus compute the two expressions for our favorite parameter values. We find that *the average land rent paid by unskilled is higher in a doughnut city than in gentrified city*, which sounds plausible because these workers are closer to the CBD and commute every workday. Despite the fact that the urban costs borne by the unskilled are lower in a doughnut city, these workers pay a higher land rent. This effect is due to their reallocation closer to CBD which allows them to significantly save on commuting costs. The corresponding drop in commuting costs exceeds the hike in land rent, and thus the net effect for the unskilled is positive because they borne lower urban costs. Besides the income effect described above, this shows that telecommuting affects the unskilled through a variety of channels.

Summarizing, we have:

Proposition 5. *If $\rho > \bar{\rho}$, then the central urban area is inhabited by the unskilled while the skilled move to the city periphery. As the WFH share increases, urban costs diminish for both skilled and unskilled workers. Furthermore, the average housing expenditure made by the skilled is lower in a doughnut city than that in a gentrified city while the opposite holds for the unskilled.*

Finally, in a doughnut city, the skilled land gradient $-(1-\rho)\xi w_s L_s$ is always smaller than the unskilled land gradient $-\xi w_\ell$. Hence, like in a gentrified city, the land rent has an outward kink at the border between the two groups of workers.

5 Inter-city commuting

We now consider an economy with two cities $i = 1, 2$. Each city is defined by a one-dimensional space X_i with a dimensionless CBD located at $0 \in X_i$. We denote a location and its distance to the CBD by $x_i \in X_i$. The two CBDs are connected by a transportation link that allows people to travel from any center to the other. For simplicity, we assume that the two cities accommodate the same masses of skilled and unskilled workers, $L_{s1} = L_{s2} \equiv L_s$ and $L_{\ell 1} = L_{\ell 2} \equiv L_\ell$. However, cities differ in their total factor productivity.

The production function of the final sector in city i is given by

$$F_i(L_s, \lambda_c) = \epsilon_i (A(\rho) L_s)^\beta \lambda_c^{1-\beta},$$

where $\epsilon_i > 0$ is city i's TFP. We assume that city 1 is more productive than city 2, i.e., $\epsilon_1 > \epsilon_2$. Consequently, wages are higher in city 1 than in city 2.

We assume that skilled workers are mobile between cities while the unskilled are not. That is, the skilled can reside in one city and work in the other. When compared to the one-city model studied above, the distinctive feature of inter-city commuting is that *the mass of skilled workers in a city differs from the mass of skilled residents*. Likewise, the gross product of the more (resp., less) productive city is higher (resp., lower) than the total income of its residents. Our goal is to study how different intensities of WFH affect the internal structure of the two cities, the inter-city commuting pattern and the wage ratios.

There are two types of inter-city commuting. First, since wages are higher in city 1 than in city 2, urban costs are also higher in city 1. Thus, skilled workers of city 1 may find it desirable to relocate in city 2 while remaining employed in city 1 (Brueckner *et al.*, 2021). This raises urban costs in city 2 and lowers those in city 1. We call these workers *between-city commuters*. Wages are unaffected by the residential shift because the labor pools do not change. Hence, between-city commuters reside in city 2 only. Second, telecommuting allows workers to enjoy a wider range of job opportunities. Indeed, as city 1 is more productive than city 2, city 2's skilled workers may want to take a job in city 1 where wages are higher while keeping their residence in city 2. We call such workers *between-city workers.* In both settings, the skilled who live and work in the same city are called *within-city commuters.* In the next two sections, we consider the case where cities are such that no skilled wants to change her residence or workplace from one city to the other in the absence of telecommuting. In this case, inter-city (if any) arises because there is telecommuting.

Adding an amenity differential between the two cities poses no specific difficulties. If the more productive city has more amenities than city 2, this will reduce the mass of between-city commuters and the relocation of workers will start for a higher WFH share. On the other hand, if the less productive city is endowed with more amenities than city 1, the flow of between-commuters will start earlier and will be larger. By contrast, as between-city workers do not change their residential location, an amenity differential has no impact on the flow of between-city workers.

6 Residential migration to the less productive city

Let $\alpha \geq 0$ be the share of city 1's skilled workers who choose to reside in city 2, i.e., they live in city 2 and work in city 1. In this case, the masses of skilled workers living in cities 1 and 2 are, respectively, equal to $(1-\alpha)L_s$ and $(1+\alpha)L_s$. The commuting cost between the two CBDs is denoted $\xi_c w_{s1} > 0$.

We assume that between-city commuters pass through the CBD of city 2 and, then, go to the CBD of city 1. Since commuting costs are proportional to wages, the between-city commuter who lives at x_2 in city 2 pays the intra-city commuting cost $(1-\rho)\xi w_{s1} x_2$ and the between-city commuting cost $(1-\rho)\xi_c w_{s1}$. The skilled who work and reside in city 2 bear a commuting rate equal to ξw_{s2} while the unskilled have a commuting rate equal to $\xi w_{\ell 2}$.

Wages in city $i = 1, 2$ are as follows:

$$w_{si} = \epsilon_i \beta \left(A(\rho) L_s\right)^{\beta} \left(\boldsymbol{L} - \eta(1-\rho)\right)^{1-\beta}, \qquad w_{\ell i} = \frac{1-\beta}{\beta} \frac{w_{si}}{\boldsymbol{L} - \eta(1-\rho)}. \tag{23}$$

Since the labor force in city 1 remains the same, between-city commuting does not affect how this city is organized. That is, the social structure of city 1 is still determined by the condition $\rho \lesseqgtr \bar{\rho}$ where $\bar{\rho}$ is defined by (16).

In contrast, the social organization of city 2 is affected by inter-city commuting. Since $w_{s1} > w_{s2}$ for all $\rho \in [0,1]$, between-city commuters always outbid within-city commuters, so that the former locate closer to the CBD than the latter regardless of the internal structure of city 2. However, the unskilled may set up at the outskirts of city 2, in between the in-between and within-city commuters or near the CBD. City 2 may then display three different social structures according to the WFH share.

(i) When $\rho < \bar{\rho}$, the two cities are gentrified. Indeed, $\bar{\rho}$ is independent of the mass of city 1's residents. In both cities, the skilled are located closer to the CBD than the unskilled. As the between-city commuters are always closer to the CBD than the within-city commuters, the social structure of city 2 is as follows: the between-city commuters locate in $[0, \alpha L_s)$, the within-city commuters in $(\alpha L_s, (1+\alpha)L_s)$, whereas the unskilled set up in the periphery $((1+\alpha)L_s, (1+\alpha)L_s + L_\ell]$.

(ii) The social structure of cities changes when ρ is larger than $\bar{\rho}$. Indeed, Proposition 4 implies that the unskilled outbid the within-city commuters in both cities, so that the former are now closer to the CBD than the latter in both cities. Furthermore, it follows from (13) that the between-city commuters outbid the unskilled if and only if

$w_{s1}/w_{\ell 2} > 1/(1-\rho)$. Using (23), we may rewrite this condition as follows:

$$\eta\rho^2 + (\boldsymbol{L} - 2\eta)\,\rho - \left(\boldsymbol{L} - \eta - \frac{\epsilon_2}{\epsilon_1}\frac{1-\beta}{\beta}\right) < 0. \tag{24}$$

Observe that the only difference with (15) is the shifter $\epsilon_2/\epsilon_1 < 1$. Therefore, the convex parabola is shifted downward. This implies that (24) holds if $\rho < \hat{\rho}$ where $\hat{\rho} > \bar{\rho}$ is the value of ρ where the parabola intersects the horizontal axis. As a result, when $\rho \in (\bar{\rho}, \hat{\rho})$, the unskilled locate closer to the CBD than the within-city commuters while the between-city commuters still secure the central locations because the WFH share is not sufficiently high for them to choose remote locations.

Since the value of the left-hand side of (24) is positive at $\rho = 1$, $\hat{\rho} < 1$ always holds. Furthermore, lowering ϵ_2/ϵ_1 shifts (24) down, and thus $\hat{\rho}$ increases. In other words, a larger productivity gap between cities sustains the coexistence of a gentrified and productive city with a less productive doughnut city for a wider range of WFH shares.

(iii) Last, for $\rho > \hat{\rho}$, the two cities have a doughnut structure because the unskilled outbid the between-city commuters who remain closer to city 2's CBD than the local within-city commuters because they pay the commuting cost between the two CBDs.

Since the level of urban costs in city 2 varies with its internal structure, we must study separately these three configurations to determine the share of city 1's skilled who choose to live in city 2.

6.1 Small WFH shares

Assume that $0 \leq \rho < \bar{\rho}$. Similarly to (17), the urban costs of an unskilled who resides at $x_2 \in ((1+\alpha)L_s, (1+\alpha)L_s + L_\ell)$ in city 2 are given by

$$UC_{\ell 2}(x_2) = R_{\ell 2}(x_2) + \xi w_{\ell 2} x_2 = \xi w_{\ell 2}[(1+\alpha)L_s + L_\ell] \equiv UC_{\ell 2},$$

which increases with α because more skilled workers makes the average unskilled commuting distance longer.

Let w (resp., b) be the index used for the within-city (resp., between-city) commuters who reside in city 2. We show in Appendix that the urban costs borne by a between-city commuter located at $x_2 \in (0, \alpha L_s)$ in city 2 is given by

$$UC_{b2}(\alpha L_s) = \xi w_{\ell 2} L_\ell + (1-\rho)\xi w_{s2} L_s + (1-\rho)\xi w_{s1} \alpha L_s \equiv UC_{b2}, \tag{25}$$

which increases with α because of the average skilled commuting distance is longer.

Using (19) shows that the urban costs of a skilled in city 1 at any location $x_1 \in (0, (1-\alpha)L_s)$ are as follows:

$$UC_{w1} = \xi w_{\ell 1} L_\ell + (1-\rho)\xi w_{s1}(1-\alpha)L_s. \tag{26}$$

Hence, a larger share α decreases the skilled urban costs in city 1 because of the average commuting distance is shorter. As a result, the difference between urban costs in the two cities decreases when α increases.

In the absence of between-city commuting, urban costs in city 1 are higher than in city 2 due to wage difference between cities. Since the consumption good is costlessly tradable between cities, its price is equal to 1 in both cities. As a result, the cost-of-living difference between two cities arises only due to variation in urban costs. Thus, city 1's skilled workers choose to reside in city 2 and keep working in city 1 if the difference in urban costs exceeds the between-city commuting cost, that is,

$$\frac{1}{1-\rho}\,UC_{s1}\big|_{\alpha=0} - \frac{1}{1-\rho}\,UC_{b2}\big|_{\alpha=0} > \xi_c w_{s1}.$$

Since urban costs are proportional to wage, what matters for between-city commuting to emerge is whether the share of income spent on between-city commuting per office hour ξ_c is smaller than the difference between urban costs per office hour. Plugging (23) into (25) and (26) and setting $\alpha = 0$, the latter inequality takes the form

$$\frac{\epsilon_1 - \epsilon_2}{\epsilon_1}\bar{s} > \xi_c,$$

where $\bar{s}$ is the share of income spent on urban costs per office hour, which is defined by

$$\bar{s} \equiv \xi L_s \left[\frac{1-\beta}{\beta}\frac{1}{1-\rho}\frac{\boldsymbol{L}}{\boldsymbol{L} - \eta(1-\rho)} + 1\right]. \tag{27}$$

which increases with ρ if $\eta < \boldsymbol{L}/2$. This inequality is satisfied under (7) when $\beta < 1/2$, which seems empirically plausible.

The threshold ρ_0 beyond which between-city commuting arises is such that the difference between urban costs is equal to the between-city commuting cost:

$$\frac{\epsilon_1 - \epsilon_2}{\epsilon_1}\bar{s} = \xi_c. \tag{28}$$

Therefore, using (27) shows that the threshold ρ_0 is positive and smaller than $\bar{\rho}$ if and only if the following inequalities hold:

$$\frac{\epsilon_1 - \epsilon_2}{\epsilon_1}\left(1 + \frac{1-\beta}{\beta}\frac{\boldsymbol{L}}{\boldsymbol{L} - \eta}\right) < \frac{\xi_c}{\xi L_s} < \frac{\epsilon_1 - \epsilon_2}{\epsilon_1}\left(1 + \boldsymbol{L}\right), \tag{29}$$

which hold for a non-degenerated set of values if and only if (7) holds.

As mentioned in Section 5, the first inequality in (29) implies that no city 1's skilled worker wants to reside in city 2 when there is no telecommuting ($\rho = 0$). Otherwise the difference in urban costs would be sufficiently large for some skilled workers in city 1 to reside in city 2, thus implying that $\alpha > 0$ even when $\rho = 0$. The first inequality holds true when the difference in city productivities is small, the between-city commuting is expensive, or both. The second inequality in (29) holds if the difference in city productivities is not too small, the between-city commuting is not too expensive, or both. In sum, by assuming that (29) holds, we rule out the extreme cases in which city productivities are very similar or very different. Likewise, between-city commuting is assumed to be neither very cheap nor very expensive.

When there is between-city commuting, the equilibrium value of α must satisfy the equilibrium condition $UC_{s1} - UC_{c2} = (1-\rho)\xi_c w_{s1}$. Plugging (25) and (26) into this equality yields

$$\xi(w_{\ell 1} - w_{\ell 2})L_\ell + (1-\rho)\xi(w_{s1} - w_{s2})L_s - 2(1-\rho)\xi w_{s1}\alpha L_s = (1-\rho)\xi_c w_{s1},$$

which pins down the equilibrium share α^* of skilled who move to city 2. Using (23), we obtain

$$\alpha^*(\rho) = \frac{\epsilon_1 - \epsilon_2}{2\epsilon_1}\left[1 + \frac{1-\beta}{\beta}\frac{1}{1-\rho}\frac{\boldsymbol{L}}{\boldsymbol{L} - \eta(1-\rho)}\right] - \frac{\xi_c}{2\xi L_s}, \tag{30}$$

which shows that *the share of between-city commuters increases with the WFH share.*

As expected, if between-city commuting becomes cheaper ($\xi_c \downarrow$), perhaps because a high-speed railway is built between the two cities, the equilibrium share α^* increases. A higher within-city commuting rate ξ also raises α^* because it widens the gap between cities' urban costs. For the same reason, α^* increases with the productivity difference ϵ_1/ϵ_2 between cities. Last, α^* rises with the mass of skilled workers in both cities ($\boldsymbol{L} \downarrow$) because the average within-city commuting rate grows faster in city 1 than in city 2, which exacerbates the urban costs difference between cities. Last, α^* increases with η because a higher consumption of CBD services raises urban costs in city 1, which incentivizes more skilled to shift to city 2.

We now use consensus values of the parameters to obtain quantitative approximations for the main variables. Combes *et al.* (2019) estimate urban costs for French cities before the COVID pandemic when the WFH share was quite low. They find that within-city commuting costs account for almost 13% of their expenditure and housing for about 33%. Thus, urban costs stand for about 45% of their income.[5] Thus, when $\rho = 0$, urban costs borne by within-city commuters are given by $UC_s = \xi w_\ell L_\ell + \xi w_s L_s = 0.45 w_s$. This expression allows us to uncover the value of ξ for different plausible values of $\boldsymbol{L}$. We assume that between-city commuting does not exceed 10% of the skilled wage ($\xi_c \leq 0.1$).

For $\boldsymbol{L} = 3$ and $\epsilon_1/\epsilon_2 = 1.1$, between-city commuting arises for all $\rho \in (0, \bar{\rho})$ if the between-city commuting rate ξ_c belongs to the interval $(0.04, 0.05)$, which means that between-city commuting must be fairly inexpensive. For larger productivity difference $\epsilon_1/\epsilon_2 = 1.2$, we find $\xi_c \in (0.075, 0.092)$. In this case, for $\xi_c = 0.075$ between-city commuting arises together with WFH while the share of between-city commuters α^* rises to 0.06 when ρ approaches $\bar{\rho} = 0.37$. In other words, 6% of the skilled population of city 1 relocate to city 2 when the skilled work about 2 days per week from home.

For $\boldsymbol{L} = 4$ and $\epsilon_1/\epsilon_2 = 1.1$, we obtain $\xi_c \in (0.041, 0.068)$. For larger productivity difference $\epsilon_1/\epsilon_2 = 1.2$, we get $\xi_c \in (0.075, 0.125)$. For an between-city commuting rate equal to 7.5, α^* takes on the value 0.17 when ρ approaches $\bar{\rho} = 0.58$. In other words, 17% of the skilled population of city 1 find it desirable to relocate to city 2 when they work home 3 days a week.

6.2 Intermediate WFH shares

We now study what happens when $\bar{\rho} < \rho < \hat{\rho}$. In this case, we have seen that the within-city commuters reside at city 2's periphery. The urban cost paid by a skilled in city 2 who resides at $x_2 \in (\alpha L_s + L_\ell, (1+\alpha)L_s + L_\ell)$ is then given by

$$UC_{w2}(x_2) = R_{w2}(x_2) + (1-\rho)\xi w_{s2} x_2 = (1-\rho)\xi w_{s2}\left[(1+\alpha)L_s + L_\ell\right] \equiv UC_{w2},$$

which implies that the land rent paid by within-city commuters is $R_{w2}(x_2) = (1-\rho)\xi w_{s2}[(1+\alpha)L_s + L_\ell - x_2]$.

Since the land rent at the border between within-city commuters and unskilled is $R_{w2}(\alpha L_s + L_\ell) = (1-\rho)\xi w_{s2} L_s = R_{\ell 2}(\alpha L_s + L_\ell)$, the urban cost paid by an unskilled who resides at $x_2 \in (\alpha L_s, \alpha L_s + L_\ell)$ in city 2 is equal to

$$UC_{\ell 2}(x_2) = R_{\ell 2}(x_2) + \xi w_{\ell 2} x_2 = (1-\rho)\xi w_{s2} L_s + \xi w_{\ell 2}(\alpha L_s + L_\ell) \equiv UC_{\ell 2},$$

while the land rent she pays is $R_{\ell 2}(x_2) = (1-\rho)\xi w_{s2} L_s + \xi w_{\ell 2}(\alpha L_s + L_\ell - x_2)$.

The land rent at the border between the between-city commuters and the unskilled is then $R_{\ell 2}(\alpha L_s) = (1-\rho)\xi w_{s2} L_s + \xi w_{\ell 2} L_\ell = R_{b2}(\alpha L_s)$. Therefore, the urban cost paid by a between-city commuter who resides at $x_2 \in (0, \alpha L_s)$ in city 2 is equal to

$$UC_{b2}(x_2) = R_{b2}(x_2) + (1-\rho)\xi w_{s1} x_2 = (1-\rho)\xi w_{s2} L_s + \xi w_{\ell 2} L_\ell + (1-\rho)\xi w_{s1} \alpha L_s \equiv UC_{b2}.$$

[5] Similar numbers are reported in the US (Bureau of Transportation Statistics, 2013).

As shown by (19), the urban costs paid by the skilled in city 1 are as follows:

$$UC_{s1} = (1-\rho)\xi w_{s1}[(1-\alpha)L_s + L_\ell].$$

Skilled workers choose to reside in city 2 and to work in city 1 if $(UC_{s1} - UC_{b2})/(1-\rho) > \xi_c w_{s1}$ holds. Using (23), urban costs per office hour are now given by

$$\frac{UC_{s1}}{1-\rho} = \hat{s}_1 w_{s1}, \qquad \frac{UC_{b2}}{1-\rho} = \hat{s}_2 w_{s2}, \tag{31}$$

where

$$\hat{s}_1 = \xi L_s(1-\alpha+\boldsymbol{L}), \qquad \hat{s}_2 = \xi L_s\left[1 + \frac{1-\beta}{\beta}\frac{1}{1-\rho}\frac{\boldsymbol{L}}{\boldsymbol{L}-\eta(1-\rho)} + \alpha\frac{\epsilon_1}{\epsilon_2}\right]$$

are the shares of income spent on land and commuting per office hour in city 1 and 2. While $\hat{s}_2$ increases with ρ, the corresponding share $\hat{s}_1$ in city 1 is independent of ρ because the skilled in city 1 locate at the periphery of the city. Thus, $(UC_{s1} - UC_{b2})/(1-\rho)$ decreases with ρ.

The equilibrium share α^* of between-city commuters is pined down by $(UC_{s1} - UC_{b2})/(1-\rho) = \xi_c w_{s1}$. Using (23) and (31), we obtain α^*:

$$\alpha^* = \frac{\epsilon_1 - \epsilon_2}{2\epsilon_1} + \frac{1}{2}\left[1 - \frac{\epsilon_2}{\epsilon_1}\frac{1-\beta}{\beta}\frac{1}{1-\rho}\frac{1}{\boldsymbol{L}-\eta(1-\rho)}\right]\boldsymbol{L} - \frac{\xi_c}{2\xi L_s}. \tag{32}$$

Hence, *the share of between-city commuters now decreases with the WFH share* (if $\eta < \boldsymbol{L}/2$). The same holds when η rises. These two results differ from what we obtained in the case where $\rho < \bar{\rho}$. On the other hand, the effects of a change in ξ_c, ξ, ϵ_1/ϵ_2, and L_s are the same as in the case of low WFH shares.

Substituting $\bar{\rho}$ into (30) and (32) yields the same value given by

$$\alpha^*|_{\rho=\bar{\rho}} = \frac{\epsilon_1 - \epsilon_2}{2\epsilon_1}(1+\boldsymbol{L}) - \frac{\xi_c}{2\xi L_s}. \tag{33}$$

Consequently, α^* is continuous at $\bar{\rho}$.

Since α^* now decreases over $(\bar{\rho}, \hat{\rho})$, the highest share α^* of between-city commuters is reached at $\rho = \bar{\rho}$ and given by (33). If (33) is non-positive, then there is no between-city commuting for all $\rho < \hat{\rho}$. Since our main focus is on between-city commuting, we assume that (33) is positive, that is,

$$\frac{\xi_c}{\xi L_s} < \frac{\epsilon_1 - \epsilon_2}{\epsilon_1}(1+\boldsymbol{L}). \tag{34}$$

This condition holds when productivity difference between cities is not too low, the between-city commuting rate relative to the within-city commuting rate is not too high, or both.

The share of inter-city commuters takes its lowest value over this domain at $\rho = \hat{\rho}$:

$$\alpha^*|_{\rho=\hat{\rho}} = \frac{\epsilon_1 - \epsilon_2}{2\epsilon_1} - \frac{\xi_c}{2\xi L_s}. \tag{35}$$

Two cases may arise. In the first one, if $\alpha^*|_{\rho=\hat{\rho}} < 0$, then there exists a value of $\rho_1 \in (\bar{\rho}, \hat{\rho})$ beyond which α^* is equal to 0. In the second case, if $\alpha^*|_{\rho=\hat{\rho}} > 0$, then $\alpha^* > 0$ over $(\bar{\rho}, \hat{\rho})$.

To gain further insights, we compute α^* for our preferable set of parameters. When $\boldsymbol{L} = 3$, $\epsilon_1/\epsilon_2 = 1.2$ and $\xi_c = 0.075$, the highest share of between-city commuters $\alpha^* = 0.06$ is reached at $\bar{\rho} = 0.37$. This share decreases and becomes 0 at $\rho = 0.41$, which is smaller than $\hat{\rho} = 0.5$. For $\boldsymbol{L} = 4$, we have $\alpha^* = 0.17$ at $\bar{\rho} = 0.58$. In this case, α^* takes the value 0 at $\rho = 0.62$ while $\hat{\rho} = 0.66$. In both cases, we have the first scenario. For α^* to be positive at $\hat{\rho} = 0.66$, between-city commuting must be fairly inexpensive, i.e., $\xi_c < 0.025$. Alternatively, for $\xi_c = 0.05$, α^* is positive at $\hat{\rho} = 0.73$ if the productivity difference is large, $\epsilon_1/\epsilon_2 > 1.5$.

6.3 High WFH shares

When $\rho > \hat{\rho}$, the within-city commuters locate in the suburbs of city 2, the unskilled locate next to the CBD, while the between-city commuters set up between these two groups. In this case, the urban costs paid by a within-city commuter who resides at $x_2 \in (\alpha L_s + L_\ell, (1+\alpha)L_s + L_\ell]$ in city 2 is given by

$$UC_{w2}(x_2) = R_{w2}(x_2) + (1-\rho)\xi w_{s2} x_2 = (1-\rho)\xi w_{s2}\left[(1+\alpha)L_s + L_\ell\right] \equiv UC_{w2},$$

so that the land rent that prevails at x_2 is equal to $R_{w2}(x_2) = (1-\rho)\xi w_{s2}[(1+\alpha)L_s + L_\ell - x_2]$.

At the border between the within-city and between-city commuters, the land rent is equal to $R_{w2}(\alpha L_s + L_\ell) = (1-\rho)\xi w_{s2} L_s = R_{b2}(\alpha L_s + L_\ell)$. Therefore, between-city commuters bear the urban cost

$$UC_{b2}(\alpha L_s + L_\ell) = (1-\rho)\xi w_{s2} L_s + (1-\rho)\xi w_{s1}(\alpha L_s + L_\ell) = UC_{b2}.$$

In city 1, the skilled bear urban costs given by

$$UC_{s1} = (1-\rho)\xi w_{s1}[(1-\alpha)L_s + L_\ell].$$

Solving $UC_{s1} = UC_{b2} + (1-\rho)\xi_c w_{s1}$ for α yields:

$$\alpha^* = \frac{\epsilon_1 - \epsilon_2}{2\epsilon_1} - \frac{\xi_c}{2\xi L_s}. \tag{36}$$

Thus, when ρ takes on high values, *the share of between-city commuters is independent of the WFH share.* Clearly, $\alpha^* > 0$ if and only if

$$\frac{\xi_c}{\xi L_s} < \frac{\epsilon_1 - \epsilon_2}{\epsilon_1}.$$

6.4 The bell-shaped curve of between-city commuting

We are now equipped to describe how the pattern of between-city commuting varies with the WFH share. Three cases may arise according to the level of the between-city commuting rate.

We have seen that α^* reaches its maximum at $\rho = \bar{\rho}$. Therefore, if (34) does not hold, α^* is always equal to zero. There is no between-city commuting for all WFH shares because between-city commuting is expensive, the productivity difference between cities is small, or both. Contrast to that, if (34) holds, there are between-city commuters for some values of the WFH share.

The following proposition provides a summary.

Proposition 6. *If* (34) *does not hold, then there is no between-city commuting regardless of the value of the WFH share. Otherwise, the equilibrium mass of between-city commuters first increases and, then, decreases with the WFH share.*

The following three patterns may arise. First, if between-city commuting and the productivity difference between cities are such that

$$\frac{\xi_c}{\xi L_s} < \frac{\epsilon_1 - \epsilon_2}{\epsilon_1}$$

holds, there is always between-city commuters regardless of the WFH share, though the value of α^* depends on ρ. In other words, inexpensive between-city commuting (perhaps because a new highway or high-speed railway is built) triggers the relocation of some city 1's skilled into city 2.

Second, if

$$\frac{\epsilon_1 - \epsilon_2}{\epsilon_1} < \frac{\xi_c}{\xi L_s} < \frac{\epsilon_1 - \epsilon_2}{\epsilon_1}\left(1 + \frac{1-\beta}{\beta}\frac{\boldsymbol{L}}{\boldsymbol{L} - \eta}\right),$$

α^* is positive for $\rho = 0$, increases up to $\rho = \bar{\rho}$ and, then, decreases and reaches the value 0 at $\rho_1 < \hat{\rho}$.

Last, if

$$\frac{\epsilon_1 - \epsilon_2}{\epsilon_1}\left(1 + \frac{1-\beta}{\beta}\frac{\boldsymbol{L}}{\boldsymbol{L} - \eta}\right) < \frac{\xi_c}{\xi L_s} < \frac{\epsilon_1 - \epsilon_2}{\epsilon_1}\left(1 + \boldsymbol{L}\right),$$

the urban pattern is richer. Indeed, $\alpha^* = 0$ over $[0, \rho_0)$ and $(\rho_1, 1]$, while α^* is bell-shaped over (ρ_0, ρ_1). Note that the last two cases correspond to between-city commuting rates which are neither very low nor very high.

As the WFH share rises, the output of the consumption sector follows the same bell-shaped pattern as the skilled wage. However, as the size of the labor pools does not change, the presence of between-city commuters does not affect the output of the consumption sector nor the wages in both cities. Thus, the wage ratio $w_1/w_2 = \epsilon_1/\epsilon_2$ is independent of the value of α. Furthermore, between-city commuters lower (resp., raise) urban costs of both skilled and unskilled workers in the more (resp., less) productive city. As wages are unaffected by between-city commuting, the new residential pattern makes the inhabitants of the less productive city worse-off. For our preferred set of parameter values, the welfare loss incurred by the unskilled in city 2 varies from 1.5 to 3.3 percent. As for the skilled, the range is $[1.5, 3.4]$. By contrast, the between-city commuters, as well as those who stay in the more productive city, are better-off. In particular, the relative gains made by the between-city commuters are comparable to the relative losses made by city 2's inhabitants: they vary from 1.7 to 3.9 percent.

Unexpectedly (at least to us), *the impact of the WFH share on the urban system is bell-shaped.* The top of the bell occurs when the social structure of the more productive city changes. Furthermore, the mass of between-city commuters is equal to 0 for small or large values of ρ. Though $\alpha^* = 0$ for low ρ seems natural, that the mass of between-city commuters is also equal to 0 when the WFH share is high is more surprising. The reason for this result is that the skilled reside in the periphery of city 1 where the land rent they pay is very low, whereas they have to pay a higher land rent in city 2 because they do not set up in this city's periphery.

7 Job migration to the more productive city

Telecommuting allows workers to enjoy a wider range of job opportunities. Indeed, when city 1 is more productive than city 2, city 2's skilled workers may want to take a job in city 1 where wages are higher while keeping their residence in city 2 where urban costs are lower. Let $\kappa \geq 0$ be the share of between-city workers. When $\kappa > 0$, the skilled labor forces in city 1 and 2 change and are, respectively, equal to $(1 + \kappa)L_s$ and $(1 - \kappa)L_s$. Therefore, a positive share κ affects wages in both cities through changes in cities' skilled labor pools, as well as in urban costs through variations in wages. Hence, between-city workers affect the urban system in a deeper way than between-city commuters.

Like the between-city commuters, between-city workers first go to the CBD of city 2 and, then, travel to the CBD of city 1. Since commuting costs are proportional to wages, the between-city workers pay the within-city commuting cost $(1 - \rho)\xi w_{s1} x_2$ and the between-city commuting cost $(1 - \rho)\xi_c w_{s1}$. The skilled who work and reside in the same city bear only the within-city commuting cost $(1 - \rho)\xi w_{si}$. All unskilled are within-commuters; their commuting rate is $\xi w_{\ell i}$.

As between-city workers affect wage schedule in cities, wages in both cities now are given by

$$w_{s1} = \epsilon_1 \beta (A(\rho)(1+\kappa)L_s)^{\beta} \left(\frac{\boldsymbol{L}}{1+\kappa} - \eta(1-\rho) \right)^{1-\beta}, \qquad w_{\ell 1} = \frac{1-\beta}{\beta} \frac{w_{s1}}{\frac{\boldsymbol{L}}{1+\kappa} - \eta(1-\rho)}, \tag{37}$$

and

$$w_{s2} = \epsilon_2 \beta (A(\rho)(1-\kappa)L_s)^{\beta} \left(\frac{\boldsymbol{L}}{1-\kappa} - \eta(1-\rho) \right)^{1-\beta}, \qquad w_{\ell 2} = \frac{1-\beta}{\beta} \frac{w_{s2}}{\frac{\boldsymbol{L}}{1-\kappa} - \eta(1-\rho)}. \tag{38}$$

In the absence of between-city workers, the skilled wage ratio is equal to the city productivity ratio ϵ_1/ϵ_2. The presence of between-city workers leads to the following skilled wage ratio

$$\frac{w_{s1}}{w_{s2}} = \frac{\epsilon_1}{\epsilon_2} \left[\frac{\frac{\boldsymbol{L}}{1+\kappa} - \eta(1-\rho)}{\frac{\boldsymbol{L}}{1-\kappa} - \eta(1-\rho)} \right]^{1-\beta} \tag{39}$$

which is lower than the productivity ratio ϵ_1/ϵ_2 when $\kappa > 0$, while the between-city unskilled wage ratio

$$\frac{w_{\ell 1}}{w_{\ell 2}} = \frac{\epsilon_1}{\epsilon_2} \left[\frac{\frac{\boldsymbol{L}}{1-\kappa} - \eta(1-\rho)}{\frac{\boldsymbol{L}}{1+\kappa} - \eta(1-\rho)} \right]^{\beta}$$

is higher than the productivity ratio. The wage ratio w_{s1}/w_{s2} decreases with the share κ of between-city workers and increases with WFH share, while the opposite holds for the ratio $w_{\ell 1}/w_{\ell 2}$.

A positive share κ of city 2's skilled workers will choose to work in city 1 if the initial skilled wage in city 1 net of between-city commuting costs and services consumption exceeds the skilled wage in city 2 net of services consumption, that is,

$$w_{s1} - (1-\rho)\eta w_{\ell 1} - (1-\rho)\xi_c w_{s1} > w_{s2} - (1-\rho)\eta w_{\ell 2}.$$

Using (37) and (38), this condition is equivalent to

$$\frac{w_{s1}}{w_{s2}} > \frac{1 - \frac{1-\beta}{\beta} \frac{\eta(1-\rho)}{\frac{\boldsymbol{L}}{1-\kappa} - \eta(1-\rho)}}{1 - \frac{1-\beta}{\beta} \frac{\eta(1-\rho)}{\frac{\boldsymbol{L}}{1+\kappa} - \eta(1-\rho)} - (1-\rho)\xi_c}. \tag{40}$$

Like in the previous section, we assume that no skilled wants to move from one city to the other in the absence of telecommuting. To obtain a necessary and sufficient condition for the absence of between-city workers when $\rho = 0$, we set $\kappa = 0$ in (40) and use (39):

$$1 - \frac{1-\beta}{\beta} \frac{\eta}{\boldsymbol{L} - \eta} < \frac{\epsilon_1}{\epsilon_1 - \epsilon_2} \xi_c. \tag{41}$$

This condition holds if the productivity difference is not too large, inter-city commuting rate is not too low, or both. We assume from now on that this condition holds. For inter-city mobility to emerge, (40) must hold for $\kappa = 0$. Plugging (39) into (40) and setting $\kappa = 0$, we obtain

$$\frac{1}{1-\rho} - \frac{1-\beta}{\beta} \frac{\eta}{\boldsymbol{L} - \eta(1-\rho)} > \frac{\epsilon_1}{\epsilon_1 - \epsilon_2} \xi_c. \tag{42}$$

The left-hand side of this inequality increases with the WFH share and goes to infinity as $\rho \to 1$ while the right-hand side in independent of ρ. Therefore, (41) implies that there exists a unique threshold value $\rho_c \in (0,1)$ such that $\kappa > 0$ for $\rho > \rho_c$.

Let κ^* be the equilibrium share of skilled in city 2 who commute to city 1. Hence, the inequality (40) holds as an equality for $\kappa = \kappa^*$. Using (39), κ^* is then pinned down by the following equation:

$$\frac{\epsilon_1}{\epsilon_2}\left[\frac{\frac{\boldsymbol{L}}{1+\kappa}-\eta(1-\rho)}{\frac{\boldsymbol{L}}{1-\kappa}-\eta(1-\rho)}\right]^{1-\beta}=\frac{1-\frac{1-\beta}{\beta}\frac{\eta(1-\rho)}{\frac{\boldsymbol{L}}{1-\kappa}-\eta(1-\rho)}}{1-\frac{1-\beta}{\beta}\frac{\eta(1-\rho)}{\frac{\boldsymbol{L}}{1+\kappa}-\eta(1-\rho)}-(1-\rho)\xi_c}. \tag{43}$$

The left-hand side is $\epsilon_1/\epsilon_2 > 1$ at $\kappa = 0$, decreases with $\kappa \in [0, 1]$ and goes to zero as κ approaches 1. The right-hand side is smaller than 1 at $\kappa = 0$, increases with κ and is larger than 1 at $\kappa = 1$. Then, this equation has a unique solution $\kappa^*(\rho)$, which is smaller than 1. Furthermore, the left-hand side is shifted upwards with ρ while the right-hand side is shifted downwards. Therefore, κ^* is an increasing function of ρ. In other words, unlike the share of between-city commuters which is non-monotone in ρ, *the share of between-city workers increases with the WFH share*. The two types of inter-city commuting have, therefore, very different impacts on cities.

We rely on our preferred set of parameters to provide a numerical illustration. For $\boldsymbol{L} = 3$, $\xi_c = 0.075$, and $\epsilon_1/\epsilon_2 = 1.1$, between-city working arises for all $\rho > \rho_c = 0.32$. As ρ goes to 1, the share of inter-city commuters rises to $\kappa^* = 0.08$. For a larger productivity gap $\epsilon_1/\epsilon_2 = 1.2$, we have $\rho_c = 0.13$ and $\kappa^* = 0.15$ when $\rho = 1$. For $\boldsymbol{L} = 4$, $\xi_c = 0.075$, and $\epsilon_1/\epsilon_2 = 1.1$, we get $\rho_c = 0.23$ while lowering inter-city commuting costs to $\xi_c = 0.05$ yields $\rho_c = 0.04$.

Finally, note the main difference between the two types of inter-city commuters. Between-city commuters lead to lower urban costs for all within-city commuters in the more productive city whose size shrinks. The opposite holds in the less productive city. Wages and output of the consumption sector remain the same in both cities. Contrast to that, between-city workers diminishes the skilled wage and rises the unskilled wage in the more productive city. The opposite holds in the less productive city. Thus, although the city sizes do not change, urban costs borne by skilled within-city commuters fall and increase for unskilled within-city commuters in the more productive city. The opposite holds in the other city. Furthermore, between-city workers shrink (resp., widen) the skilled (resp., unskilled) wage gap between cities and increase (resp., decrease) the output of the consumption sector in the more (resp., less) productive city. The two types of inter-city commuters thus shape the urban and wage structures in very different ways.

8 Concluding remarks and extensions

We have shown that telecommuting has consequences that go way beyond productivity gains or loses for firms and workers. Telecommuting has implications for the unskilled who do not work home, for the social structure of cities and for housing expenditures. For example, the renewal of urban centers observed in several big cities is likely to be negatively affected by a growing adoption of teleworking. As adopting WFH represents a fundamental shift in how firms do business, it is premature to predict that telework will trigger the great dispersion of skilled labor. Yet, unless firms choose the strategy "return to the office," telework should foster some dispersion of the skilled who also face a wider range of residential opportunities. In addition, WFH raises income inequality between the skilled and the unskilled, which may give rise to very contrasted city structures. All of this shows that WFH has spatial effects that do not occupy center stage in the on-going debates about the good and the bad of home working.

That said, one should keep in mind that our findings have been obtained under restrictive assumptions. First, for analytical simplicity, we have assumed that workers consume a fixed lot size. This clashes with Stanton and

Tiwari (2021) who find that prior to the pandemic, wired workers spend 7% more on housing than similar non-remote households in the same commuting zone. Hence, our model should be extended for housing consumption to be endogenous and increasing in the WFH share. At the other extreme of the spectrum, we consider a dimensionless CBD while telecommuters allow firms to use less office space, thus reducing real-estate costs which are high in cities where land and housing are very expensive. In other words, land should be an additional input of the final sector. We expect more land-intensive firms to lean more toward a spatial dispersion of jobs. Second, WFH conflicts with agglomeration economies. Although our model suffices to show that the presence of such economies slows down the dispersion of skilled workers, it would be interesting to study how different types of agglomeration economies may affect the city structure when they are combined with telecommuting. For example, location-dependent knowledge spillovers or the sharing of local public goods such as transit are likely to interact in different ways with telecommuting.

Third, we have combined home labor and office labor through a simple technological relationship. In the absence of compelling results regarding the interactions between these two types of labor, this strikes us as a reasonable modeling strategy. However, if WFH is going to stick, we will know more about this relationship in the near future. This should permit studying richer ramifications in the production process. Fourth, we have considered homogeneous firms and workers. Assuming that both types of agents are heterogeneous in their preferences for WFH may lead to cities that will substantially vary according to their specialization and the institutional environment in which they operate. A firm may be efficient in a certain WFH context, but not necessarily in all of them. Likewise, workers' choices to live and work in a city will depend on the labor contract they will be offered. Finally, and more importantly, how the WFH share will be chosen should have an impact on cities. However, it is far from being obvious that agents involved in the decision process will take this impact into account in their final choice.

References

[1] Adams-Prassl, A., T. Boneva, M. Golin, and C. Rauh (2020) Work that can be done from home: Evidence on variation within and across occupations and industries. IZA DP No. 13374.

[2] Allen, T.D. Golden and K.M. Shockley (2015) How effective is telecommuting? Assessing the status of our scientific findings. *Psychological Science in the Public Interest* 16: 40-68.

[3] Althoff, L., F. Eckert, S. Ganapati, and C. Walsh (2021) The geography of remote work. NBER Working Paper No. 29181.

[4] Barrero, J.M., N. Bloom and S.J. Davis (2020) Why working from home will stick. University of Chicago, Becker Friedman Institute for Economics Working Paper No. 2020-174.

[5] Bartik, A.W., Z.B. Cullen, E.L. Glaeser, M. Luca, and C.T. Stanton (2020) What jobs are being done at home during the Covid-19 crisis? Evidence from firm-level surveys. NBER Working Paper 27422.

[6] Battiston, D., J. Blanes i Vidal and T. Kirchmaier (2021) Face-to-face communication in organizations. *Review of Economic Studies* 88: 574-609.

[7] Behrens, K., S. Kichko and J.-F. Thisse (2021) Working from home: Too much of a good thing? CEPR Discussion Paper DP15669.

[8] Bloom, N. (2020) How working from home works out. Stanford, Policy Brief.

[9] Bloom, N., J. Liang, J. Roberts, and Z.J. Ying (2015) Does working from home work? Evidence from a Chinese experiment. *Quarterly Journal of Economics* 130: 165-218.

[10] Bureau of Transportation Statistics (2013) Transportation Statistics Annual Report 2013. Washington,DC: US Government Printing Office.

[11] Brueckner, J., M. Kahn and G.C. Lin (2021) A new spatial hedonic equilibrium in the emerging work–from home economy? NBER Working Paper 28526.

[12] Claudel M., E. Massaro, P. Santi, F. Murray, and C. Ratti (2017) An exploration of collaborative scientific production at MIT through spatial organization and institutional affiliation. *PLoS ONE* 12: 1-22.

[13] Combes, P.-P. and L. Gobillon (2015) The empirics of agglomeration economies. In: *Handbook of Regional and Urban Economics. Volume 5*, edited by G. Duranton, J.V. Henderson, and W.C. Strange, 247-348. Amsterdam: Elsevier.

[14] Combes, P.-P., G. Duranton and L. Gobillon (2019) The costs of agglomeration: House and land prices in French cities. *Review of Economic Studies* 86: 1556-89.

[15] Couture, V. and J. Handbury (2020) Urban revival in America. *Journal of Urban Economics* 119: 103267.

[16] Davis, M.A., A.C. Ghent and J.M. Gregory (2021) The work-from-home technology boon and its consequences. NBER Working Paper 28461.

[17] De Fraja, G., J. Matheson and J. Rockey (2021) Zoomshock: The geography and local labour market consequences of working from home. CEPR Discussion Paper DP15655.

[18] Delventhal, M., E. Kwon and A. Parkhomenko (2021) How do cities change when we work from home? *Journal of Urban Economics* (forthcoming).

[19] Dingel, J. and B. Neiman (2020) How many jobs can be done at home? *Journal of Public Economics* 189: 104235.

[20] Edlund, L., C. Machado and M.M. Sviatschi (2015) Bright minds, big rent: Gentrification and the rising returns to skill. NBER Working Paper w21729.

[21] Fujita, M. (1989) *Urban Economic Theory. Land Use and City Size*. Cambridge: Cambridge University Press.

[22] Goldfarb, A. and C. Tucker (2019) Digital economics. *Journal of Economic Literature* 57: 3-43.

[23] Gupta, A., V. Mittal, J. Peeters and S. Van Nieuwerburgh (2021) Flattening the curve: Pandemic-induced revaluation of urban real estate. *Covid Economics* 69: 1-45.

[24] Koren, M. and R. Peto (2020) Business disruptions from social distancing. *Covid Economics* 2: 13-31.

[25] Koster, P.R. and H.R.A. Koster (2015) Commuters' preferences for fast and reliable travel: A semiparametric estimation approach. *Transportation Research Part B: Methodological* 81: 289-301.

[26] Liu, S. and Y. Su (2021) The impact of the COVID-19 pandemic on the demand for density: Evidence from the U.S. housing market. Available at SSRN 3661052.

[27] Mattana, E., V. Smeets and F. Warzynski (2020) Changing skill structure and COVID-19. *Covid Economics* 45: 1-30.

[28] Mas, A. and A. Pallais (2020) Alternative work arrangements. *Annual Review of Economics* 12: 631-58.

[29] Milgrom, P. and J. Roberts (1990) The economics of modern manufacturing: Technology, strategy, and organization. *American Economic Review* 80: 511-28.

[30] Moretti, E. (2012) *The New Geography of Jobs*. Boston: Houghton Mifflin Harcourt.

[31] Morikawa, M. (2020) Productivity of working from home during the COVID-19 pandemic: Evidence from an employee survey. *Covid Economics* 49: 123-139.

[32] Rhee, H.-J. (2008) Home-based telecommuting and commuting behavior. *Journal of Urban Economics* 63: 198-216.

[33] Safirova, E. (2002) Telecommuting, traffic congestion, and agglomeration: A general equilibrium model. *Journal of Urban Economics* 52: 26-52.

[34] Small, K.A. (2012) Valuation of travel time. *Economics of Transportation* 1: 2-14.

[35] Stanton, C. and P. Tiwari (2021) Housing consumption and the cost of remote work. NBER Working Paper 28483.

Appendix

Similarly to (17), the urban costs of an unskilled who resides at $x_2 \in ((1+\alpha)L_s, (1+\alpha)L_s + L_\ell)$ in city 2 are given by

$$UC_{\ell 2}(x_2) = R_{\ell 2}(x_2) + \xi w_{\ell 2} x_2 = \xi w_{\ell 2}((1+\alpha)L_s + L_\ell) \equiv UC_{\ell 2}.$$

Since $R_{w2}((1+\alpha)L_s) = R_{\ell 2}((1+\alpha)L_s) = \xi w_{\ell 2} L_\ell$ at the border between the unskilled and the with-city commuters, the urban costs UC_{n2} of a within-city commuter is as follows:

$$UC_{w2}((1+\alpha)L_s) = \xi w_{\ell 2} L_\ell + (1-\rho)\xi w_{s2}(1+\alpha)L_s = R_{w2}(x_2) + (1-\rho)\xi w_{s2} x_2 \equiv UC_{n2}.$$

Therefore, the land rent paid by a within-city commuter at $x_2 \in (\alpha L_s, (1+\alpha)L_s)$ is equal to

$$R_{w2}(x_2) = R_{w2}((1+\alpha)L_s) + (1-\rho)\xi w_{s2}((1+\alpha)L_s - x_2),$$

so that the land rent at the border αL_s between within-city and between-city commuters is given by

$$R_{w2}(\alpha L_s) = R_{w2}((1+\alpha)L_s) + (1-\rho)\xi w_{s2} L_s = R_{b2}(\alpha L_s).$$

This yields the urban costs borne by between-city commuters at any location $x_2 \in (0, \alpha L_s)$, that is,

$$UC_{b2}(\alpha L_s) = R_{b2}(\alpha L_s) + (1-\rho)\xi w_{s1}\alpha L_s = \xi w_{\ell 2} L_\ell + (1-\rho)\xi w_{s2} L_s + (1-\rho)\xi w_{s1}\alpha L_s \equiv UC_{b2}.$$