Optimal supply-side fiscal policies for Covid-19

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What are the optimal supply-side fiscal policies to respond to the Covid-19 shock? In this work, we try to answer this question. We develop a model which embeds the technological response of firms to a pandemic shock. We propose a novel formulation of the latter, whose persistence is endogenous and dependent on the level of contact-intense activities in the economy. This paves the way for the analysis of supply side fiscal policies that allow to mitigate both the economic crisis and the spread of the disease.
The main results indicate how a contraction in overall economic activity is optimal in order to mitigate the spread of the virus; in addition, income taxation can be seen as a proxy for lockdowns. Moreover, a supply-side incentive provided to boost technological reallocation can help to contain the economic recession. Finally, we provide evidence that modelling the Covid-19 pandemic as an adverse productivity shock fails in reproducing the dynamics of the disease actually observed.

Keywords: Coronavirus, Macroeconomics, Sectoral Substitution, Fiscal Policy

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I. Introduction

We propose a simple macroeconomic model that endogenises the technological response of firms to a ”pandemic” shock. This paves the way for the analysis of supply side fiscal policies that allow to mitigate both the economic crisis and the spread of the disease.
Our model introduces the possibility for firms to switch between physical and online retail trade. Firms sell their output through two different technologies: a physical and an online one; the former requires to be contact-intensive and reproduces the fact that individuals need to physically meet in order to produce and consume some goods; the latter, instead, can be interpreted as a proxy for less contact-intensive production activities.
To characterize the pandemic in our heuristic model, we follow *Buera et. al (2021)* who characterize the shock as an increase in the utility from leisure, but in our framework the persistence of the shock is endogenous and dependent on the level of contact-intense activities in the economy.

Similar to *Uhlig et. al (2020)*, the market response to the shock entails a reallocation of labor and consumption towards the physical retail sector, and a global slowdown. This is consistent with empirical evidence documented in *Leibovici et. al (2020) Toxvaerd (2020)* and *Farboodi et. al (2020).*

This endogenous market response is the consequence of decentralized actions that do not fully internalize their implications for the pandemic persistence. We therefore consider Ramsey-optimal fiscal policies, where the planner relies on two tools, a production subsidy and an income tax rate. The first tool is obviously geared towards the optimal reallocation of production towards the online retail sector. The second one is used to slowdown economic activity in an attempt to further mitigate the pandemic. In a way, the income tax tool should also be seen as a proxy for administrative ”lockdown” policies.

Our analysis therefore allows to identify both the optimal sectoral reallocation and the optimal economic contraction in response to the shock. As stated by *Loayza and Pennings (2020)* and *Dupor (2020)*, a macroeconomic stimulus aimed at propelling aggregate demand may not necessary be the best tool in the middle a containment effort, i.e. when the policy maker's goal is avoiding the spread of the disease.

We contribute to the strand of heuristic models literature, concerned with the modeling of the pandemic in relatively simple non-SIR macroeconomic models.
So far, the Covid-19 shock has been treated as a combination of demand and supply factors.
Guerreri et. al (2020) propose a stylized economic model to analyse whether a supply side shock (i.e. a shutdown shock reducing the participation in the labor market) can trigger a negative demand effect in sectors other than the one initially affected. They claim that these Keynesian supply shocks may take place only in a multi-sector environment characterised by complementarity among goods. *Corrado et. al (2021)* model the Covid crisis as a combination of standard demand, supply and technology shocks.

In *Faria-e-Castro (2020),* Covid-19 is associated to a large negative demand shock, obtained through a decrease in the utility of consumption. The framework is exploited to investigate the effectiveness of different fiscal policies in stabilising the economy during the pandemic. *Buera et. al (2021)* show how the speed and magnitude of the post-shock recovery depends on the duration of the lockdown and on the public policies supporting individuals during the shutdown. Both works investigate the role played by demand side policies, such as an increase in government consumption and in unemployment insurance benefits, but cannot provide insights on the importance of supply-side policies. Our contribution is different because the endogenous persistence mechanism introduced here captures the need to limit overall economic activity in order to mitigate the spread of the pandemic. Our model therefore endogeneises the trade-off between economic and economic impacts of individual and policy decisions. In the paper we also illustrate why characterizing the pandemic with standard shocks is inappropriate for the purpose of welfare and optimal policy analysis. Further research will be devoted to incorporating our supply side characterisation into a macro-SIR model (*Eichenbaum et al. (2020)* among the others), in order to have a more accurate description of the evolution of the pandemic, alongside with the analysis of the public policies required to mitigate it.

II. The model

The economy is characterised by the presence of four agents: households, intermediate firms, final firms and the public sector. The model embeds price stickiness and labor reallocation frictions.
Households consume, hold their wealth through government bonds and supply differentiated labor services. Intermediate firms produce intermediate inputs for the final good firms, operating in a perfectly competitive environment.
Final producers are monopolistically competitive and face price rigidities; they can exploit two different technologies to produce their output, a physical and an online one.
The public sector provides two different types of subsidy: the first is aimed at offsetting the distortion involved by the presence of imperfect competition in the final market, while the second is destined to the online production channel, in order to incentive its exploitation, following the occurrence of the shock. Moreover, the government levies labor income taxation and issues public debt. Finally, the economy is hit by a labor disutility shock, whose magnitude depends on the level of physical interaction required by the different economic activities.

A. Intermediate Firms

The intermediate sector is characterised by the presence of fully competitive firms producing intermediate goods which will be used as production inputs by the final sector.

Firms have access to the following production function:

$$S_{I,t} = AN_{I,t}^{\alpha} \tag{1}$$

where $S_{I,t}$ is the intermediate output, $N_{I,t}$ is the labor used as productive factor in the intermediate production and A defines the level of productivity. Intermediate firms are subject to decreasing returns to scale, as $\alpha < 1$.
In each period, firms maximise their profits:

$$\Pi_{I,t} = p_{I,t}S_{I,t} - w_{I,t}N_{I,t} \tag{2}$$

where $p_{I,t}$ is the relative price of the intermediate good and $w_{I,t}$ is the real wage paid to workers in the intermediate sector.
The solution of the problem provides the optimal demand for intermediate labor:

$$w_{I,t} = p_{I,t}\alpha AN_{I,t}^{\alpha-1} \tag{3}$$

B. Final Firms

The final good sector is composed by a number j of producers, which operate in a monopolistically competitive market and face nominal rigidities in the form of sticky prices.
In order to produce their output, final firms can access two different production functions:

$$S_{rf,t}^{j} = \left[\left(\frac{N_{rf,t}^{j}}{\tau_{rf}^{j}}\right)^{\alpha_r}\left(S_{Irf,t}^{j}\right)^{1-\alpha_r}\right]^{\theta} \tag{4}$$

$$S_{ro,t}^{j} = \left[\left(\frac{N_{ro,t}^{j}}{\tau_{ro}^{j}}\right)^{\alpha_r}\left(S_{Iro,t}^{j}\right)^{1-\alpha_r}\right]^{\theta} \tag{5}$$

where the first equation refers to the physical production function and the second to the

online one.
$S^j_{rf,t}$ and $S^j_{ro,t}$ are respectively the physical and the online outputs of the firm. Moreover, the aggregate final output S^j_t is composed by the sum of the physical and the online outputs:

$$S^j_t = S^j_{rf,t} + S^j_{ro,t} \tag{6}$$

$N^j_{rf,t}$ and $N^j_{ro,t}$ are the quantities of physical and online workforce needed by the firms, while $\tau^j_{rf,t}$ and $\tau^j_{ro,t}$ are the related production costs.
Final firms acquire inputs from the intermediate sector and exploit them in both technologies ($S^j_{Irf,t}$ and $S^j_{Iro,t}$).
The parameter α_r indicates the share of labor involved in the final production and $\theta < 1$ considers decreasing returns to scale in both technologies.

Price rigidities

Price stickiness is modelled à la Rotemberg (1982). In each period firms can choose the optimal price subject to a quadratic adjustment cost:

$$C = \frac{\gamma}{2}\left(\frac{P^j_t}{P^j_{t-1}} - 1\right)^2 S_t \tag{7}$$

Thus, the problem final firms have to face is:

(8)
$$\Pi^j_t = \frac{P^j_t}{P_t} S^j_t - (1-\nu_t)\left(w^j_{ro,t} N^j_{ro,t} + p_{I,t} S^j_{Iro,t}\right) - w^j_{rf,t} N^j_{rf,t} - p_{I,t} S^j_{Irf,t} - \frac{\gamma}{2}\left(\frac{P^j_t}{P^j_{t-1}} - 1\right)^2 S_t$$

st

$$S^j_{rf,t} = \left[\left(\frac{N^j_{rf,t}}{\tau^j_{rf}}\right)^{\alpha_r} (S^j_{Irf,t})^{1-\alpha_r}\right]^{\theta} \tag{9}$$

$$S^j_{ro,t} = \left[\left(\frac{N^j_{ro,t}}{\tau^j_{ro}}\right)^{\alpha_r} (S^j_{Iro,t})^{1-\alpha_r}\right]^{\theta} \tag{10}$$

$$S_t^j = S_{rf,t}^j + S_{ro,t}^j \tag{11}$$

$$S_t^j = S_t \left(\frac{P_t^j}{P_t} \right)^{-\psi} \tag{12}$$

where the first two constraints are the physical and the online production functions, the third is the aggregation rule for the final output and the fourth is the demand the for final good faced by each firm. Thus, $(\frac{P_t^j}{P_t})$ is the relative price for good j and ψ is the price elasticity of demand.

Final producers receive from the public sector a subsidy ν_t, whose aim is providing an incentive to a larger utilisation of the online technology. As a matter of fact, as the strength of the shock is a positive function of the level of physical interaction in the economy (as observed in reality due to the pandemic), the policy maker's goal consists in reducing the intensity of physical activities. In order to obtain this shift in the production function, an incentive is given to the online retail of final production.

Firms optimally choose $N_{rf,t}^j, N_{ro,t}^j, S_{Irf,t}^j, S_{Iro,t}^j, P_t^j$ so that the first order conditions of the problem with respect to labors and intermediate inputs are:

$$N_{rf,t}^j = \frac{\theta \alpha_r MC_t^j S_{rf,t}^j}{w_{rf,t}^j} \tag{13}$$

$$N_{ro,t}^j = \frac{\theta \alpha_r MC_t^j S_{ro,t}^j}{(1-\nu_t) w_{ro,t}^j} \tag{14}$$

$$S_{Irf,t}^j = \frac{\theta (1-\alpha_r) MC_t^j S_{rf,t}^j}{p_{I,t}} \tag{15}$$

$$S_{Iro,t}^j = \frac{\theta (1-\alpha_r) MC_t^j S_{ro,t}^j}{(1-\nu_t) p_{I,t}} \tag{16}$$

where the first two are the demands for labors, while the others are the demands for intermediate inputs.
Moreover, the solution with respect to the price yields the New Keynesian Phillips Curve:

$$(1-\psi)+\psi(1-\omega_t)MC_t+\gamma\mathbb{E}_t\left[(\pi_{t+1}-1)\pi_{t+1}\frac{S_{t+1}}{S_t}\right]=\gamma(\pi_t-1)\pi_t\frac{S_{t+1}}{S_t} \tag{17}$$

According to the last equation, inflation today, π_t, depends on the expectation of tomorrow inflation, π_{t+1}, and on the level of marginal cost of production. The term ω_t directly affects the firm marginal cost and it is a public subsidy provided to offset the distortion implied by monopolistic competition. More specifically, it operates in the direction of eliminating the presence of the markup over the price, a standard feature of imperfect markets.
Hence, the steady state value of the marginal cost looks as:

$$MC=\left(\frac{\psi-1}{\psi}\right)\frac{1}{(1-\omega)} \tag{18}$$

Hence, by setting:

$$\omega=\frac{1}{\psi} \tag{19}$$

the effect of the markup is eliminated as if the market would be characterised by perfect competition, i.e. the marginal cost would be equal to the price.

MARGINAL COST

The marginal cost faced by final firms can be derived from both physical and online total costs; this yields:

$$MC_{rf,t} = \frac{1}{\theta}(1-\nu_t)(\tau_{rf})^{\alpha_r}\left(\frac{w_{rf,t}}{\alpha_r}\right)^{\alpha_r}\left(\frac{P_t^I}{(1-\alpha_r)}\right)^{1-\alpha_r}(S_{rf,t})^{\frac{1-\theta}{\theta}} \tag{20}$$

and

$$MC_{ro,t} = \frac{1}{\theta}(1-\nu_t)(\tau_{ro})^{\alpha_r}\left(\frac{w_{ro,t}}{\alpha_r}\right)^{\alpha_r}\left(\frac{P_t^I}{(1-\alpha_r)}\right)^{1-\alpha_r}(S_{ro,t})^{\frac{1-\theta}{\theta}} \tag{21}$$

However, it must hold that the physical and the online marginal costs are equal for the firm. In addition, by considering the fact that the final output is the sum of the online and the physical outputs, the total marginal cost can be written as:

$$MC_t = \frac{1}{\theta}(1-\nu_t)(\tau_{ro})^{\alpha_r}\left(\frac{w_{ro,t}}{\alpha_r}\right)^{\alpha_r}\left(\frac{P_t^I}{(1-\alpha_r)}\right)^{1-\alpha_r}\left(\frac{S_t}{\left[1+\left[(1-\nu_t)\frac{(\tau_{ro})^{\alpha_r}}{\left(\tau_{rf}\right)^{\alpha_r}}\left(\frac{w_{ro,t}}{w_{rf,t}}\right)^{\alpha_r}\right]^{\frac{\theta}{1-\theta}}\right]}\right)^{\frac{1-\theta}{\theta}} \tag{22}$$

C. *Households*

Households preferences are defined over consumption S_t and labor effort, which can be divided in three different types: intermediate $N_{I,t}$, physical final $N_{rf,t}$ and online final $N_{ro,t}$. The representative household's lifetime utility function is akin to Moura (2018):

$$E_0\sum_{t=0}^{\infty}\beta^t\left\{\frac{(S_t)^{1-\sigma}}{1-\sigma}-\frac{1+\alpha_t^N}{1+k}\left[\chi_1(N_{rf,t})^{1+\eta}+\chi_2(N_{ro,t})^{1+\eta}+\chi_3(N_{I,t})^{1+\eta}\right]^{\frac{1+\kappa}{1+\eta}}\right\} \tag{23}$$

Where β is the discount factor, σ is the intertemporal elasticity of substitution, α^N is the labor disutility shock. The specification of the labor bundle implies reallocation rigidities, and hence imperfect labor mobility, when η is larger than zero. This, in principle, would introduce heterogeneity in wages and hours worked. The parameter κ measures the aggregate elasticity of labor supply and χ_1, χ_2 and χ_3 are weights attached respectively to the physical, online and intermediate labor.

The budget constraint is:

$$P_tS_t + R_{t-1}B_{t-1} = B_t + (1-t_t)(W_{rf,t}N_{rf,t} + W_{ro,t}N_{ro,t} + W_{I,t}N_{I,t}) + \Pi_t \tag{24}$$

Where P_t is the nominal price, B_{t-1} is the stock of government bond the household holds, R_{t-1} is the nominal interest rate, B_t is purchase of public bonds, $W_{rf,t}$, $W_{ro,t}$ and $W_{I,t}$ are nominal wages paid for, respectively, physical ($N_{rf,t}$), online ($N_{ro,t}$) and intermediate ($N_{ro,t}$) labor. Finally, t_t are distortionary income taxes and Π_t are firms profits.
The households' first order conditions are:

$$\lambda_t = \frac{(S_t)^{-\sigma}}{P_t} \tag{25}$$

$$\lambda_t\beta^t = \lambda_{t+1}\beta^{t+1}R_t \tag{26}$$

(27)

$$\chi_1(1+\alpha^N)\left[\chi_1(N_{rf,t})^{1+\eta} + \chi_2(N_{ro,t})^{1+\eta} + \chi_3(N_{I,t})^{1+\eta}\right]^{\frac{\kappa-\eta}{1+\eta}}(N_{rf,t})^{\eta} = \lambda_t(1-t_t)W_{rf,t}$$

(28)

$$\chi_2(1+\alpha^N)\left[\chi_1(N_{rf,t})^{1+\eta} + \chi_2(N_{ro,t})^{1+\eta} + \chi_3(N_{I,t})^{1+\eta}\right]^{\frac{\kappa-\eta}{1+\eta}}(N_{ro,t})^{\eta} = \lambda_t(1-t_t)W_{ro,t}$$

(29)

$$\chi_3(1+\alpha^N)\left[\chi_1(N_{rf,t})^{1+\eta} + \chi_2(N_{ro,t})^{1+\eta} + \chi_3(N_{I,t})^{1+\eta}\right]^{\frac{\kappa-\eta}{1+\eta}}(N_{I,t})^{\eta} = \lambda_t(1-t_t)W_{I,t}$$

D. Public sector

The public sector provides two different subsidies: ω_t, that is meant to remove the distortion implied by the presence of monopolistic competition in the final good market; and ν_t, which is provided to the online retail of final producers.
The government finances ω_t with lump sum taxes, while ν_t is financed through the aforementioned distortionary taxation on the wage bill, t_t.
t_t moves according to the following rule:

$$t_t = \iota\left(\frac{B_{t-1}}{B}\right)^{\xi} \tag{30}$$

where ι is the sensitivity of taxation to the debt level and ξ defines the intensity of the reaction of taxation to debt accumulation. Taxes react to deviations of the public debt to its steady state value, with a one period ahead lag.
The government's budget constraint is:

$$v_t + \frac{R_{t-1}b_{t-1}}{\pi_t} = t_t(w_{rf,t}N_{rf,t} + w_{ro,t}N_{ro,t} + w_{I,t}N_{I,t}) + b_t \tag{31}$$

where b_t is public debt (in real terms) and π_t is the inflation level. Hence, the public sector needs to levy taxes on the labor income and to issue new debt in order to repay interest on past debt and to finance the provision of the online subsidy.

E. Monetary policy

The monetary authority is assumed to conduct a monetary policy by adjusting the nominal interest rate based on the Taylor rule that targets the deviation of inflation and output from their respective steady-state level:

$$\frac{R}{\bar{R}} = \left(\frac{\pi_t}{\bar{\pi}}\right)^{\theta_\pi} \left(\frac{St}{\bar{S}}\right)^{\theta_S} \tag{32}$$

F. The shock

The economy is hit by the following labor disutility shock:

$$\alpha_t^N = \left(\alpha_{t-1}^N\right)^\rho \left(\frac{N_{rf,t}}{N_{rf,t} + N_{ro,t} + N_{I,t}}\right)^{\Delta(1-\rho)} \exp \varepsilon_t \tag{33}$$

where ρ is the persistence of the process and Δ defines the intensity of the shock as a function of the level of physical interaction in the economy. The aim of this structure is mimicking the effect of the Covid-19 pandemic and this creates the need of a shock incorporating:

- an increase in the disutility of labor, i.e. households should be less willing to supply their labor as a consequence of the pandemic;
- the magnitude of the impact should be related to the frequency of inter-personal interaction for physical production and consumption.

The second point concerns the peculiar nature of the pandemic shock, given that its strength is increasing in the physical quantity produced, reproducing a more active participation to the physical production (and so, an higher probability of infection). This is obtained through the presence of the ratio $(\frac{N_{rf,t}}{N_{rf,t}+N_{ro,t}+N_{I,t}})$, which can be interpreted as the probability of getting infected. Hence, the higher this probability (due to a high level of physical production), the stronger the shock and the consequent labor destruction.

III. Model calibration

The steady state values are calibrated on the U.S. economy on the eve of the pandemic shock. The calibration is summarised in Table 1.
Most households parameters are standard, with the exception of those related to the presence of frictions in the labor market, κ and η. They are both assumed to be equal to 2, following the calibration proposed in *Moura (2018)*.
With respect to production and labor markets, the share of labor in the production function is assumed to be $\alpha = \alpha_r = 0.66$ both for intermediate and final productions.
The ratio $\frac{Srf}{Sro}$ is equal to 4 and it is calibrated according to data from the U.S. Census Bureau; it replicates the fact that physical production is four times larger than the online one.
The share of the different labors are calibrated such that the physical labor accounts for the 65% of total labor, while online and intermediate are, respectively, 15% and 20%. These calibrations follow *OECD (2021)*.
Final firms are subject to decreasing returns to scale; hence, the value of θ implied by the model is 0.8754.
The parameter of the sensitivity of taxation with respect to debt, ι, is chosen to have a steady state debt to GDP annualised ratio equal to 60%.
Monetary policy parameters are reasonably standard, with θ_π and θ_S which are respectively 1.5 and 0.2.

(i) Parameter	Value	Definition
β	0.99	Households discount factor
σ	0.9	Elasticity of intertemporal substitution
κ	2	Aggregate elasticity of labor supply
η	2	Reallocation cost for labor
α	0.66	Share of labor in intermediate production
α_r	0.66	Share of labor in final production
θ	0.8754	Returns to scale final production
γ	18.5	Rotemberg menu cost
ψ	6	Price elasticity of demand
ι	0.008	Sensitivity of taxation to debt
ξ	0.9	Intensity of tax reaction to debt accumulation
θ_π	1.5	Taylor rule: inflation
θ_S	0.2	Taylor rule: output
Δ	16	Shock intensity
ρ	0.9	Shock persistence

(ii) Steady State	Value	Definition
$\frac{Srf}{Sro}$	4	Ratio physical to online output
N_{rf}	0.65	Share of physical labor on total labor
N_{ro}	0.15	Share of online labor on total labor
N_I	0.2	Share of intermediate labor on total labor
$\frac{B}{S}$	0.6	Annualised debt to GDP ratio

Table 1—: *(i) Main parameters (ii) Steady state values*

IV. Results

A. Labor disutility shock

In order to investigate how the Ramsey planner would intervene in response to the pandemic, we introduce our proposal for the Covid-19 pandemic shock, that is designed as a labor disutility shock, whose strength is an increasing function of the level of physical interaction required by the production:

$$\alpha_t^N = \left(\alpha_{t-1}^N\right)^{\rho} \left(\frac{N_{rf,t}}{N_{rf,t} + N_{ro,t} + N_{I,t}}\right)^{\Delta(1-\rho)} \exp \varepsilon_t$$

The term $(\frac{N_{rf,t}}{N_{rf,t}+N_{ro,t}+N_{I,t}})$ can be considered as the probability of becoming infected. We chose to consider only the physical sector (hence, the physical labor) at the numerator, in order to underline the higher level of contagiousness relative to the online sector [1].
In our experiment, we simulate the Covid-19 shock affecting the economy and we consider two different specifications of the instruments set of the Ramsey planner, in order to analyse the optimal fiscal supply side policies. We compare this two cases with the market economy, where no public intervention is made and taxes follows a simple heuristic rule.
Figure 1 presents the results of the simulation for a shock calibrated to reproduce a 10% drop in aggregate output.
The disutility shock affects every type of labor, making the households less willing to supply their workforce. As a consequence, in the market economy scenario (green solid line), physical, online and intermediate outputs decrease; this implies the drop in aggregate output. The nature of the shock does not naturally nudge a reallocation towards the online and less contact-intensive sector. We choose to adopt this framework - the virus affects all the three labors - to better replicate a precise feature of the pandemic. Covid-19 affects labor through two different channels. On the one hand, there is a direct impact, as physical labor is more at risk of infection, because it requires more contact-intensive activities; on the other, even though some activities may be conducted remotely, their productivity could still deteriorate. Some examples could be people working from home but sharing the workplace/home with infected relatives, or experiencing a lack of technical resources required from their task.
The Ramsey planner faces two different objectives: mitigating the economic recession

[1] We simulate the same experiment with the infection probability considering also the intermediate sector. Namely, $(\frac{N_{rf,t}+N_{I,t}}{N_{rf,t}+N_{ro,t}+N_{I,t}})$. The result are in line with those obtained with the first simulation and are presented in the Appendix A

while trying to contain the spread of the disease.
The blue dashed line presents the Ramsey optimal fiscal policy with the planner controlling one instrument. The provision of a positive online subsidy is successful in achieving a reallocation of labor towards the online sector, with the physical and intermediate productions decreasing more than in the market economy case. This shift dampens the fall in aggregate output and achieve a reduction in the infection probability.
The planner's aim is exploiting the online subsidy in order to face the problem of costly labor reallocation.
The light blue dotted line shows the planner's strategy when he controls, beside the subsidy, also the level of income taxation. The dynamics of the economy are qualitatively similar, but with some crucial differences. The drop in physical labor and output is more pronounced, since in this scenario the planner is more efficient in controlling the pandemic. While the reallocation effect is still valid, the economy experiences a more severe contraction in aggregate output, due to a tighter fiscal regime; in fact, the income tax tool can be seen as a proxy for administrative lockdown policies. Hence, the increase in the level of taxation reduces the households purchasing power and consequently generates a reduction in consumption. This also implies a further decrease in the infection probability, relative to the one instrument case[2].

[2]Even though the results of the two Ramsey problems regarding the infection probability seem to be very close, note that the probability is expressed as $(\frac{N_{rf,t}}{N_{rf,t}+N_{ro,t}+N_{I,t}})$; in the two instruments case, the drop in aggregate labor (the denominator) is more pronounced, so the overall economic contraction is stronger

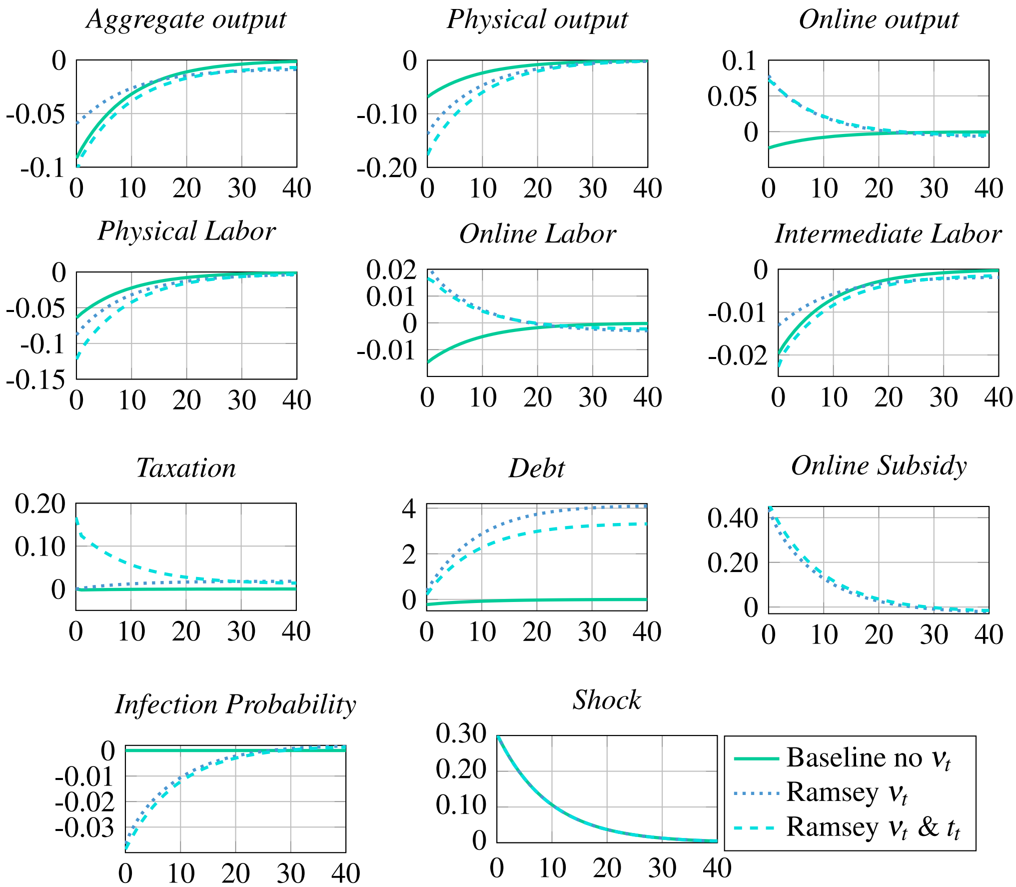

Figure 1. : *Effects of a labor disutility shock.*
Simulation of the economy hit by a disutility shock affecting all the types of labor. The figure shows the variations in levels from the steady state.

B. *Robustness: the role of labor frictions*

In this section, we evaluate the impact of labor reallocation frictions on the dynamics of the model. Thus, we test our model for different values of the parameter κ, i.e. the aggregate elasticity of labor supply. Figure 2 presents the results for the Ramsey optimal problem with one instrument[3].

[3]We perform the same analysis for the one instrument case, but the results are in line with those for one instrument. See Appendix C

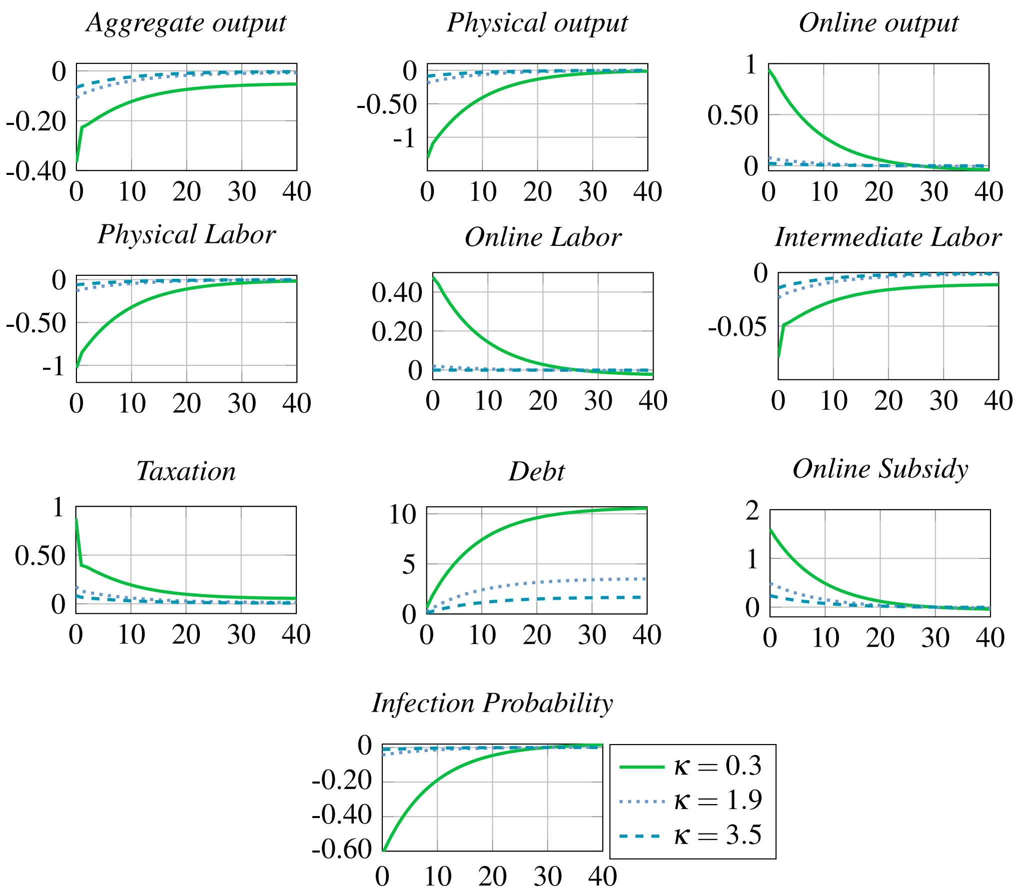

Figure 2. : Robustness check for κ, Ramsey two instruments case.
Simulation of the economy for different levels of labor elasticity. The figure shows the variations in levels from the steady state.

The results show that the presence of stronger market frictions, i.e. an higher value of κ implies a significant reduction in the strength of the reallocation mechanism. Hence, we observe that our framework clearly describes the planner's strategy to fight the pandemic, that is i) encouraging the reallocation process towards the less contagious (and more efficient) sector and ii) a tighter fiscal regimes (higher level of taxation) to achieve a stronger control over the spread of the disease. The more elastic is the labor supply, the more effective would be the reallocation process.

C. Adverse productivity shock

In this section, we present evidence that, when adopting a Ramsey-optimal approach, designing the pandemic shock uniquely as an adverse productivity shock fails in reproducing the consequences and dynamics observed in the reality.

This framework features a different specification for the shock, which is not a labor supply shock and does not depend on the level of physical interaction in the economy. Instead, physical production suffers an increase in the production cost, τ_{rf}, while the online is unaffected. The shock assumes the following form:

$$\tau_{rf,t} = (1-\rho)\bar{\tau_{rf}} + \rho\,\tau_{rf,t-1} + \varepsilon_\tau \qquad (34)$$

where ε_τ is a white noise exogenous shock to the physical production cost and ρ indicates the persistence of the shock.

The shock is calibrated in order to have a 10% drop in the aggregate output. Figure 3 presents the results for three different simulations, i) the market economy result, without any intervention of the Ramsey planner (green solid line), ii) the scenario where the Ramsey planner controls only one instrument, i.e. the online subsidy v_t (blue dashed line) and iii) the scenario where the Ramsey planner controls *two* instruments, i.e. the online subsidy v_t and the level of taxation t_t (light blue dotted line).

All simulations show how, in response to the shock, the planner's intervention produces almost the same effects of those obtained through the market mechanism; as a matter of fact, the increase in labor cost produces a contraction in the production of the physical output, because firms are less willing to employ physical labor. The possibility to shift towards the online technology partially contains the recession, but this transition mechanism is not strong enough to avoid the fall of aggregate output.

It is worth noting the response of the planner to the shock, in panels 7 and 9. The variation in the levels of the two instruments is close to zero, meaning that the planner is not willing to correct the market mechanism. Due to the frictionless labor market, labor force is free to move from the less to the more efficient technology. Hence, in this scenario, the planner and the market responses are the same, underlying the fact that the market mechanism is already the efficient one.

However, the small variations in subsidy and taxation are due to the planner's willingness to correct the inefficiency implied by the presence of price stickiness. The shock triggers an increase in the production costs and hence also the inflation rises accordingly.

The monetary policy chosen by the central authority is not tight enough to counteract the inflation, i.e. the increase in the interest rate is not high enough. Hence, the planner needs to intervene: through the negative online subsidy, in order to make this sector less productive and fight the increase in prices. Or, in the two instruments scenario, the intervention is obtained through taxation: in fact, the decreased overall productivity needs less labor effort.

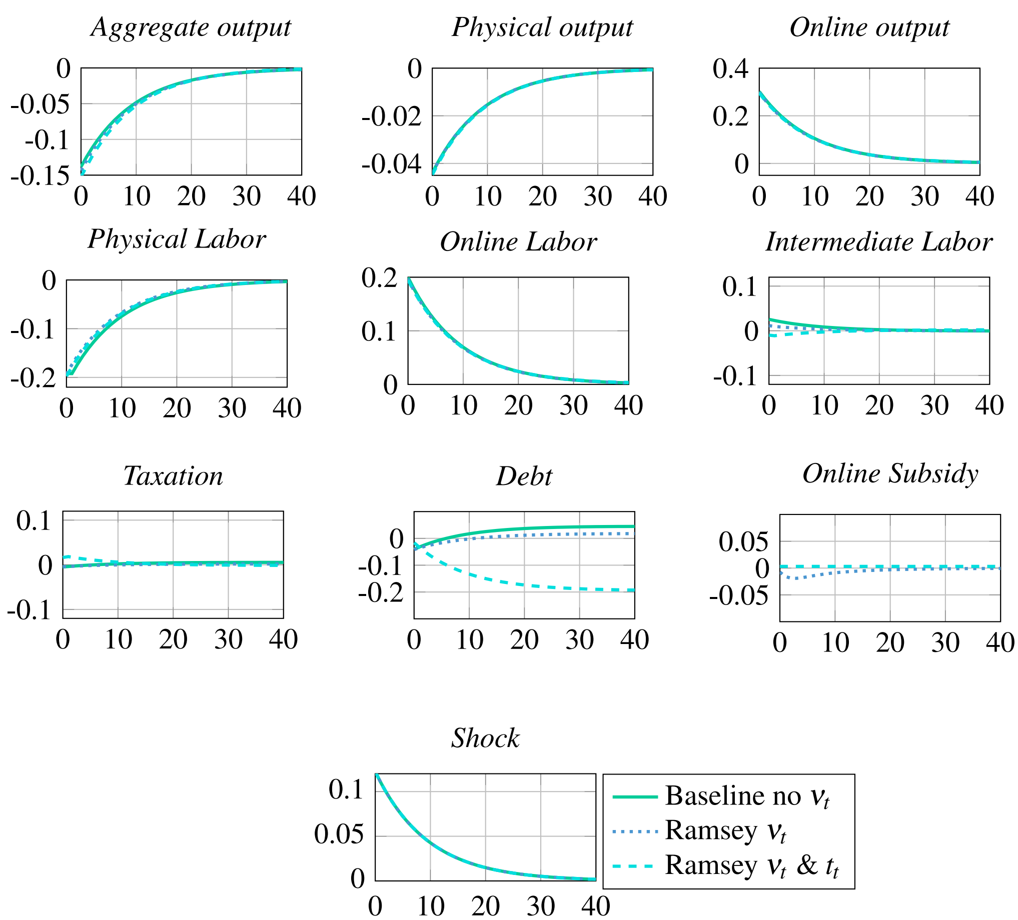

Figure 3. : *Effects of a productivity shock.*
Simulation of the economy hit by physical sector productivity shock. The figure shows the variations in levels from the steady state.

V. Conclusions

The aim of this paper was analysing the optimal supply side fiscal policies in response to the Covid-19 outbreak.
We proposed a macroeconomic model which incorporates the Covid-19 pandemic. The shock is designed as a labor disutility shock, with endogenous persistence and dependent on the level of contact-intensive activities.
We introduced the possibility for firms to exploit two different production technologies, i.e. a physical and an online one. This feature generates a shift between the two types, as a reaction to the shock.
The main result is twofold. On the first hand, when the policymaker is provided with two fiscal instruments, he achieves a reduction in the spread of the pandemic, but obtained through a drop of the aggregate output. Yet, the economic recession is partially mitigated thanks to a stronger reallocation effect towards the less contagious sector, via the online subsidy. Furthermore, the subsidy generates an incentive for firms to invest in the online channel, and it works alongside with income taxation, which can be seen a proxy for an administrative lockdown (given that it is more efficient in containing the spread of the pandemic).
On the other hand, we think that our paper contributes the literature relative to the design of the Covid-19 shock in macroeconomic models. Our representation is able to capture some of the shortfall of designing the pandemic as productivity shocks. In fact, in some cases, despite providing useful insights, this approach lacks of a proper characterisation of the dynamics observed in reality. Hence, using a different design for the shock can lead to more accurate policy implications.

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Appendix

Appendix A

As a robustness check, here a different formulation of the labor disutility shock α_N.The infection probability is a function of both physical and intermediate labors, such that:

$$\alpha_t^N = (\alpha_{t-1}^N)^\rho \left(\frac{N_{rf,t} + N_{I,t}}{N_{rf,t} + N_{ro,t} + N_{I,t}} \right)^{\Delta(1-\rho)} \exp \varepsilon_t \tag{1}$$

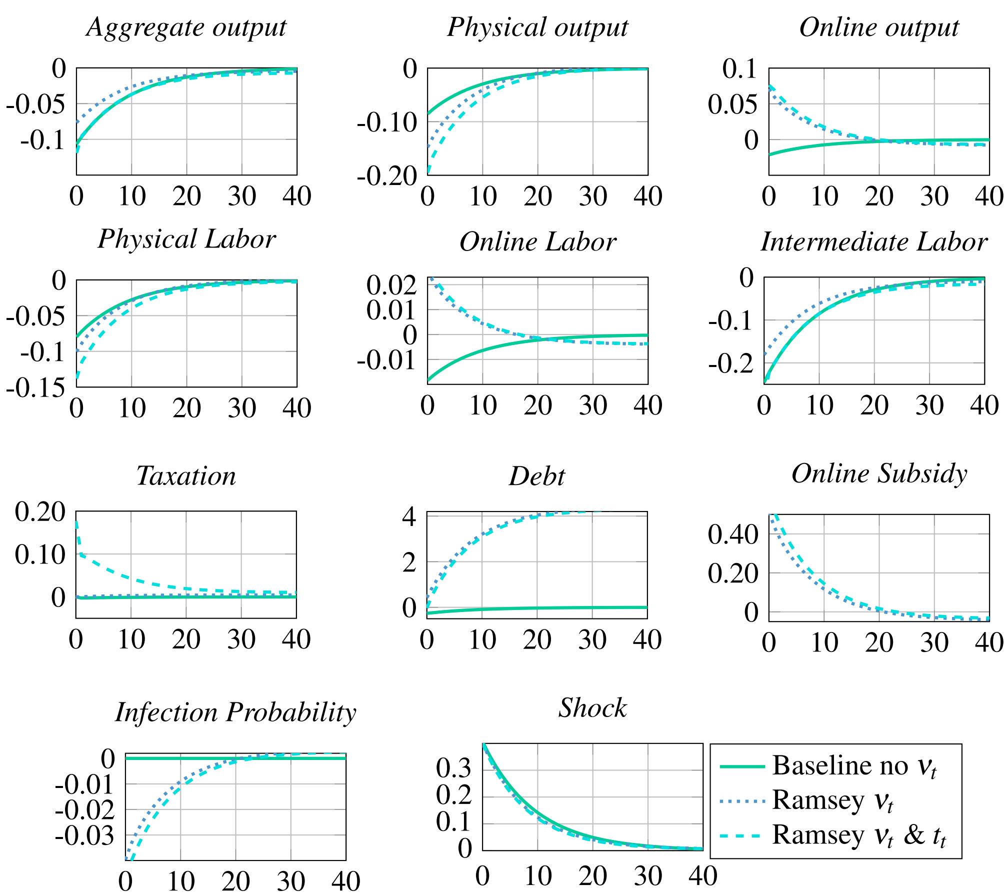

Figure 4. : *Effects of a labor disutility shock.*
Simulation of a disutility shock; infection probability considers physical and intermediate labors. Variations are in levels from the steady state.

Introducing this formulation means having a stronger impact of the pandemic on the economy, as more sectors are considered as contact-intensive, so that the infection probability is higher. As a consequence, the impact of the shock is more pronounced and hence the strength of the response of the planner is more intense, both in the case with one and two instruments.
Overall, the dynamics are the same as those described in the benchmark case considering only the physical sector in the formulation of the infection probability.

APPENDIX B

In this section we present a modification of the model more in line with the results of *Uhlig et. al (2020)*. In their model, households internalise- even in the market economy scenario- the different likelihood of being infected as a consequence of their consumption and labor choices. More precisely, they describe a scenario with different infection probabilities according to the different sector, i.e. one sector is more contact-intensive than the other.
Our framework is able to replicate this outcome as a special case. Namely, we design the shock α^N affecting only two types of labor, the physical and the intermediate. This reproduces the fact that this two sectors are more at risk of infection, relatively to the online.
Hence, the households utility function becomes:

$$(2) \qquad E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{(S_t)^{1-\sigma}}{1-\sigma} - \frac{1}{1+\kappa} \left[\begin{array}{c} \left(1+\alpha_t^N\right) \chi_1 (N_{rf,t})^{1+\eta} + \\ \chi_2 (N_{ro,t})^{1+\eta} + \left(1+\alpha_t^N\right) \chi_3 (N_{I,t})^{1+\eta} \end{array} \right]^{\frac{1+\kappa}{1+\eta}} \right\}$$

The results concerning the planner's intervention are in line with those obtained with our benchmark shock. However, it is worth noting the difference in the response of the market economy scenario; we can observe how households internalise the infection risk derived from working in physical and intermediate sectors and, consequently, decide to shift their labor towards the online retail. This generates an expansion in the online sector and a reduction in the infection probability, since the economy converges towards a less contact-intensive environment. As already said, this results are more consistent with the one presented in literature by *Uhlig et. al (2020)*.

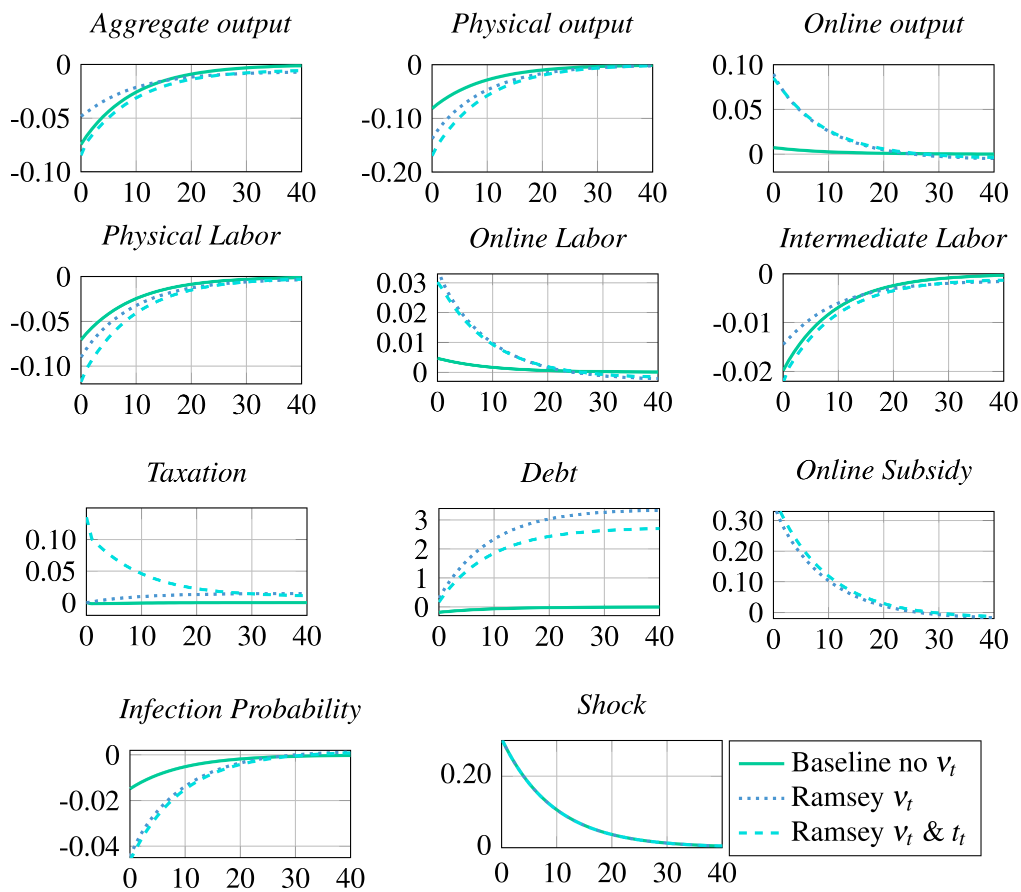

Figure 5. : *Effects of a labor disutility shock.*
Simulation of the economy hit by a shock affecting only physical and intermediate labors. The figure shows the variations in levels from the steady state.

Appendix C

This appendix presents the results for the robustness check of the parameter κ in the scenario where the planner optimally set only one instrument, i.e. the online subsidy ν_t. The results are in line with those obtained in the two instruments case: with a lower degree of labor reallocation frictions the planner can provide a larger subsidy and achieve a stronger shift from the physical to the online sector.

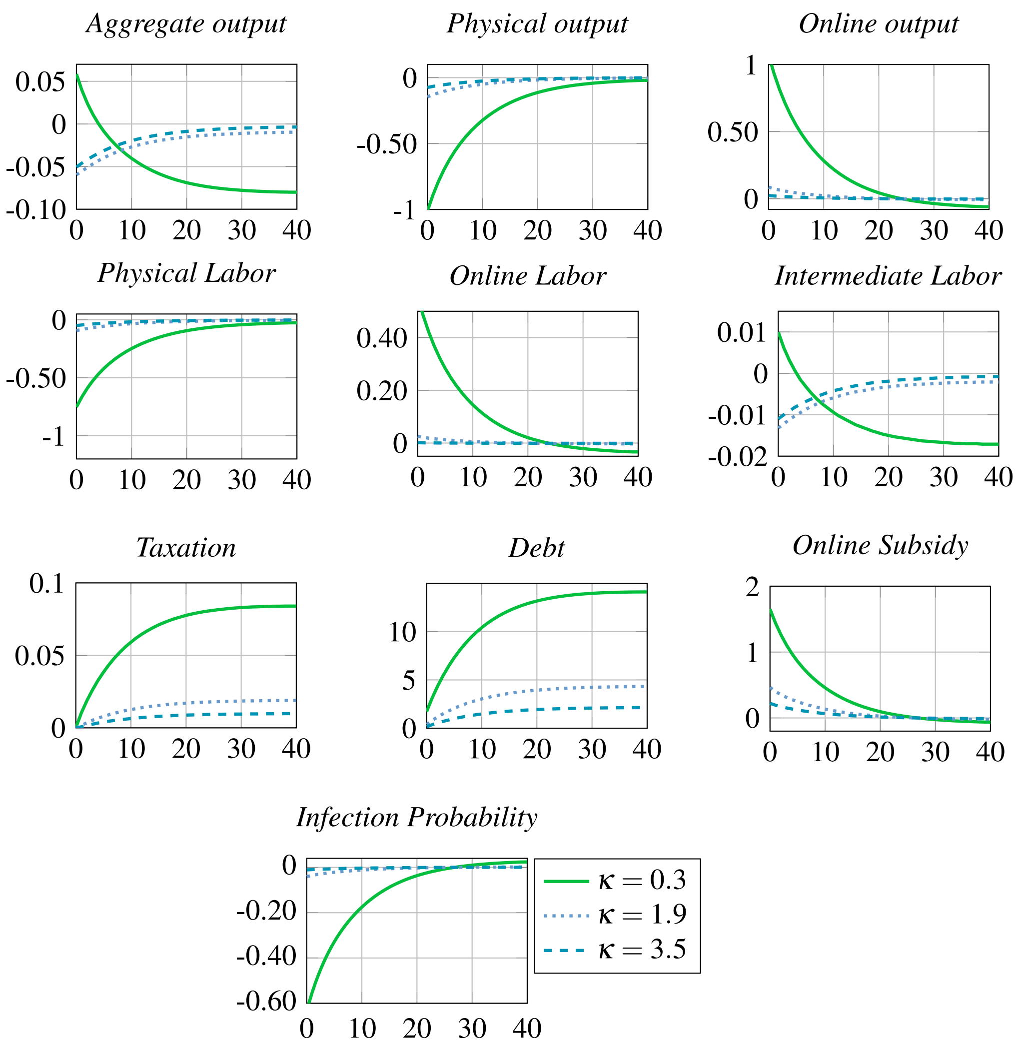

Figure 6. : *Robustness check for κ, Ramsey one instruments case.*
Simulation of the economy for different levels of labor elasticity. The figure shows the variations in levels from the steady state.

APPENDIX D

In this section the full derivation of the problems is presented

INTERMEDIATE FIRM

The problem of the intermediate firm is:

$$\max \Pi_{I,t} = p_{I,t} S_{I,t} - w_{I,t} N_{I,t} \tag{3}$$

$$st$$

$$S_{I,t} = A N_{I,t}^{\alpha} \tag{4}$$

$\frac{\partial L}{\partial S_{I,t}} = 0$

$$\lambda_t = p_{I,t}$$
$$\lambda_t = MC_{I,t}$$
$$MC_{I,t} = p_{I,t} \tag{5}$$

Through Cost Minimization, the demand for intermediate labor, $N_{I,t}$, can be found:

$$\max \Pi_{I,t} = p_{I,t} S_{I,t} - w_{I,t} N_{I,t} \tag{6}$$

$$st$$

$$S_{I,t} = A N_{I,t}^{\alpha} \tag{7}$$

$\frac{\partial L}{\partial N_{I,t}} = 0$

$$w_{I,t} = p_{I,t} \alpha A N_{I,t}^{\alpha - 1} \tag{8}$$

NB: the price p_I is the relative intermediate price, i.e. $p_I = \frac{P_I}{P_t}$, and $w_{I,t}$ is the real intermediate wage, i.e $w_{I,t} = \frac{W_{I,t}}{P_t}$.

Final Firms

The problem of the final producer is the following:

(9)
$$\max \Pi_t^j = \frac{P_t^j}{P_t} S_t^j - (1-\nu_t)\left(w_{ro,t}^j N_{ro,t}^j + p_{I,t} S_{Iro,t}^j\right) - w_{rf,t}^j N_{rf,t}^j - p_{I,t} S_{Irf,t}^j - \frac{\gamma}{2}\left(\frac{P_t^j}{P_{t-1}^j} - 1\right)^2 S_t$$

$$st$$

$$S_{rf,t}^j = \left[\left(\frac{N_{rf,t}^j}{\tau_{rf}^j}\right)^{\alpha_r} (S_{Irf,t}^j)^{1-\alpha_r}\right]^{\theta} \tag{10}$$

$$S_{ro,t}^j = \left[\left(\frac{N_{ro,t}^j}{\tau_{ro}^j}\right)^{\alpha_r} (S_{Iro,t}^j)^{1-\alpha_r}\right]^{\theta} \tag{11}$$

$$S_t^j = S_{rf,t}^j + S_{ro,t}^j \tag{12}$$

$$S_t^j = S_t \left(\frac{P_t^j}{P_t}\right)^{-\psi} \tag{13}$$

The Lagrangean of the problem is:

$$L = \frac{P_t^j}{P_t} S_t - (1-\nu_t)\left(w_{ro,t}^j N_{ro,t}^j + p_{I,t} S_{Iro,t}^j\right) - w_{rf,t}^j N_{rf,t} - p_{I,t} S_{Irf,t}^j - \frac{\gamma}{2}\left(\frac{P_t^j}{P_{t-1}^j} - 1\right)^2 S_t$$
$$-\frac{P_t^j}{P_t} MC_t^j \left[S_t^j - [(\frac{N_{rf,t}^j}{\tau_{rf}^j})^{\alpha_r} (S_{Irf,t}^j)^{1-\alpha_r}]^{\theta} - [(\frac{N_{ro,t}^j}{\tau_{ro}^j})^{\alpha_r} (S_{Iro,t}^j)^{1-\alpha_r}]^{\theta}\right] \tag{14}$$

The first order conditions are:

$\frac{\partial L}{\partial N_{rf,t}} = 0$

$$N_{rf,t}^j = \frac{\theta \alpha_r MC_t^j S_{rf,t}^j}{(1-\nu_t) w_{rf,t}^j} \tag{15}$$

$\frac{\partial L}{\partial N_{ro,t}} = 0$

(16)
$$N^j_{ro,t} = \frac{\theta\alpha_r MC^j_t S^j_{ro,t}}{(1-\nu_t)\,w^j_{ro,t}}$$

$\frac{\partial L}{\partial S_{Irf,t}} = 0$

(17)
$$S^j_{Irf,t} = \frac{\theta(1-\alpha_r)MC^j_t S^j_{rf,t}}{(1-\nu_t)\,p_{I,t}}$$

$\frac{\partial L}{\partial S_{Iro,t}} = 0$

(18)
$$S^j_{Iro,t} = \frac{\theta(1-\alpha_r)MC^j_t S^j_{ro,t}}{(1-\nu_t)\,p_{I,t}}$$

$\frac{\partial L}{\partial p_t} = 0$

(19)
$$(1-\psi)\,S_t\frac{P_t^{j-\psi}}{P_t^{1-\psi}} - \gamma\left(\frac{P^j_t}{P^j_{t-1}} - 1\right)\frac{1}{P^j_{t-1}}S_t + \gamma\left(\frac{P^j_{t+1}}{P^j_t} - 1\right)\frac{1}{P_t^{j\,2}}S_{t+1} - \psi MC^j_t S_t\frac{P_t^{j-\psi-1}}{P_t^{-\psi}} = 0$$

Consider a symmetric equilibrium where $P^j_t = P_t$ and set $\frac{P^j_t}{P^j_{t-1}} = \pi_t$

$$(1-\psi)\,S_t\frac{1}{P_t} - \psi MC_t S_t\frac{1}{P_t} - \gamma(\pi_t - 1)\frac{1}{P_{t-1}}S_t + \gamma(\pi_{t+1} - 1)\frac{1}{P_t^2}S_{t+1} = 0$$

Now multiply for P_t and divide for S_t

$$(1-\psi) - \psi MC_t - \gamma(\pi_t - 1)\pi_t + \gamma(\pi_{t+1} - 1)\pi_{t+1}\frac{S_{t+1}}{S_t} = 0$$

Hence, by considering the anti-monopolistic subsidy ω_t, we finally obtain:

(20)
$$(1-\psi) + \psi(1-\omega_t)MC_t + \gamma\mathbb{E}_t\left[(\pi_{t+1} - 1)\pi_{t+1}\frac{S_{t+1}}{S_t}\right] = \gamma(\pi_t - 1)\pi\frac{S_{t+1}}{S_t}$$

MARGINAL COST

The marginal cost faced by final firms can be derived from both physical and online total costs; this yields:

$$MC_{rf,t} = \frac{1}{\theta}(1-\nu_t)(\tau_{rf})^{\alpha_r}\left(\frac{w_{rf,t}}{\alpha_r}\right)^{\alpha_r}\left(\frac{P_t^I}{(1-\alpha_r)}\right)^{1-\alpha_r}(S_{rf,t})^{\frac{1-\theta}{\theta}} \tag{21}$$

and

$$MC_{ro,t} = \frac{1}{\theta}(1-\nu_t)(\tau_{ro})^{\alpha_r}\left(\frac{w_{ro,t}}{\alpha_r}\right)^{\alpha_r}\left(\frac{P_t^I}{(1-\alpha_r)}\right)^{1-\alpha_r}(S_{ro,t})^{\frac{1-\theta}{\theta}} \tag{22}$$

However, it must hold that the physical and the online marginal cost are equal for the firm:

$$\mathrm{MC}_{rf,t} = MC_{ro,t} \tag{23}$$

This yields:

$$\frac{S_{rf,t}}{S_{ro,t}} = \left[(1-\nu_t)\left(\frac{\tau_{ro}}{\tau_{rf}}\right)^{\alpha_r}\left(\frac{w_{ro,t}}{w_{rf,t}}\right)^{\alpha_r}\right]^{\frac{\theta}{1-\theta}}$$

Moreover, considering the relation

$$S_t = S_{rf,t} + S_{ro,t}$$

yields:

$$S_{ro,t} = \frac{S_t}{1 + \left[(1-\nu_t) \frac{(\tau_{ro,t})^{\alpha_r}}{(\tau_{rf,t})^{\alpha_r}} \left(\frac{w_{ro}}{w_{rf}} \right)^{\alpha_r} \right]^{\frac{\theta}{1-\theta}}}$$

Finally, the expression for the marginal cost is:

$$\mathrm{MC}_t = \frac{1}{\theta} (1-\nu_t) (\tau_{ro,t})^{\alpha_r} \left(\frac{w_{ro,t}}{\alpha_r} \right)^{\alpha_r} \left(\frac{P_t^I}{(1-\alpha_r)} \right)^{1-\alpha_r} \left(\frac{S_t}{\left[1 + \left[(1-\nu_t) \frac{(\tau_{ro,t})^{\alpha_r}}{(\tau_{rf,t})^{\alpha_r}} \left(\frac{w_{ro}}{w_{rf}} \right)^{\alpha_r} \right]^{\frac{\theta}{1-\theta}} \right]} \right)^{\frac{1-\theta}{\theta}}$$

(24)

HOUSEHOLDS

The households problem assumes the following form:

(25)

$$\max U_t = E_t \beta^t \left[\frac{(S_t)^{1-\sigma}}{1-\sigma} - \frac{1+\alpha_t^N}{1+\kappa} \left[\chi_1 (N_{rf,t})^{1+\eta} + \chi_2 (N_{ro,t})^{1+\eta} + \chi_3 (N_{I,t})^{1+\eta} \right]^{\frac{1+\kappa}{1+\eta}} \right]$$

st

(26) $$P_t S_t + R_{t-1} B_{t-1} = B_t + (1-t_t) \left[W_{rf,t} N_{rf,t} + W_{ro,t} N_{ro,t} + W_{I,t} N_{I,t} \right]$$

The Lagrangean of the problem is:

$$L = E_t \beta^t \left\{ \begin{array}{c} \left[\frac{(S_t)^{1-\sigma}}{1-\sigma} - \frac{1+\alpha_t^N}{1+\kappa} \left[\chi_1 (N_{rf,t})^{1+\eta} + \chi_2 (N_{ro,t})^{1+\eta} + \chi_3 (N_{I,t})^{1+\eta} \right]^{\frac{1+\kappa}{1+\eta}} \right] \\ -\lambda_t \left[P_t S_t + R_{t-1} B_{t-1} - B_t - (1-t_t) \left[W_{rf,t} N_{rf,t} + W_{ro,t} N_{ro,t} + W_{I,t} N_{I,t} \right] \right] \end{array} \right\}$$

The first order conditions are:

$I.\ \frac{\partial L}{\partial S_t} = 0$

$$\lambda_t = \frac{(S_t)^{-\sigma}}{P_t} \tag{27}$$

$II.\ \frac{\partial L}{\partial B_t} = 0$

$$-\lambda_t \beta^t + \lambda_{t+1}\beta^{t+1} R_t = 0 \tag{28}$$

$III.\ \frac{\partial L}{\partial N_{rf,t}} = 0$

(29)
$$\left(1+\alpha_t^N\right)\left[\chi_1(N_{rf,t})^{1+\eta} + \chi_2(N_{ro,t})^{1+\eta} + \chi_3(N_{I,t})^{1+\eta}\right]^{\frac{\kappa-\eta}{1+\eta}} \chi_1(N_{rf,t})^{\eta} = \lambda_t(1-t_t)W_{rf,t}$$

$IV.\ \frac{\partial L}{\partial N_{ro,t}} = 0$

(30)
$$\left(1+\alpha_t^N\right)\left[\chi_1(N_{rf,t})^{1+\eta} + \chi_2(N_{ro,t})^{1+\eta} + \chi_3(N_{I,t})^{1+\eta}\right]^{\frac{\kappa-\eta}{1+\eta}} \chi_2(N_{ro,t})^{\eta} = \lambda_t(1-t_t)W_{ro,t}$$

$V.\ \frac{\partial L}{\partial N_{I,t}} = 0$

(31)
$$\left(1+\alpha_t^N\right)\left[\chi_1(N_{rf,t})^{1+\eta} + \chi_2(N_{ro,t})^{1+\eta} + \chi_3(N_{I,t})^{1+\eta}\right]^{\frac{\kappa-\eta}{1+\eta}} \chi_3(N_{I,t})^{\eta} = \lambda_t(1-t_t)W_{I,t}$$

Plug I into II to obtain the Euler equation

$$\frac{(S_t)^{-\sigma}}{P_t} = \beta \frac{(S_{t+1})^{-\sigma}}{P_{t+1}} R_t$$

$$R_t = \frac{1}{\beta}\frac{P_{t+1}}{P_t}\left(\frac{S_t}{S_{t+1}}\right)^{-\sigma}$$

$$\frac{1}{R_t} = \beta\left[\Pi_{t+1}^{-1}\left(\frac{S_{t+1}}{S_t}\right)^{-\sigma}\right] \tag{32}$$

Plug I into III, IV and V to obtain the three labor supplies:

(33)

$$(1-t_t)w_{rf,t} = \left(1+\alpha_t^N\right)\chi_1\left[\chi_1(N_{rf,t})^{1+\eta}+\chi_2(N_{ro,t})^{1+\eta}+\chi_3(N_{I,t})^{1+\eta}\right]^{\frac{\kappa-\eta}{1+\eta}}(N_{rf,t})^{\eta}(S_t)^{\sigma}$$

and

(34)

$$(1-t_t)w_{ro,t} = \left(1+\alpha_t^N\right)\chi_2\left[\chi_1(N_{rf,t})^{1+\eta}+\chi_2(N_{ro,t})^{1+\eta}+\chi_3(N_{I,t})^{1+\eta}\right]^{\frac{\kappa-\eta}{1+\eta}}(N_{ro,t})^{\eta}(S_t)^{\sigma}$$

and

(35)

$$(1-t_t)w_{I,t} = \left(1+\alpha_t^N\right)\chi_3\left[\chi_1(N_{rf,t})^{1+\eta}+\chi_2(N_{ro,t})^{1+\eta}+\chi_3(N_{I,t})^{1+\eta}\right]^{\frac{\kappa-\eta}{1+\eta}}(N_{I,t})^{\eta}(S_t)^{\sigma}$$

NB : note that we have expressed real wages as $\frac{W_{rf,t}}{P_t}=w_{rf,t}$, $\frac{W_{ro,t}}{P_t}=w_{ro,t}$, $\frac{W_{I,t}}{P_t}=w_{I,t}$