Are Ideas Really Getting Harder To Find?

R&D Capital and the Idea Production Function*

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Abstract

We supplement the 'Idea Production Function' (IPF) with measures of R&D capital. We construct a time series of R&D capital stock in the US (1968-2019) based on cumulated R&D investment. We estimate the IPF with patent applications as R&D output, allowing for a flexible treatment of unit productivity of R&D capital and R&D labor. We find that the elasticity of substitution between R&D input factors is $0.7 - 0.8$ and significantly below unity. This implies that R&D capital is an essential factor in producing ideas, complementary to R&D labor. We also identify a systematic positive trend in R&D labor productivity at about 1% per year on average and a cyclical trend in R&D capital productivity. Our results suggest that instead of 'ideas getting harder to find', there is an increasing scarcity of R&D capital needed to find them.

Keywords: R&D, Long-Run Growth, Technical Change, Estimation, CES.

JEL Codes: O30, O40, O47

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1 Introduction

Technological change, due to purposeful R&D activities, is widely acknowledged as a fundamental driver of economic growth. Technological ideas, due to their non-rivalrous nature, essentially act as a source of increasing returns to scale allowing output to grow even when input usage is constant (Romer, 1990). However, whilst these precepts are well-established in the literature, the exact specification of the R&D process (the "Idea Production Function", hereafter IPF) is subject to considerable dispute.[1]

In particular, and perhaps surprisingly, the majority of the existing R&D-based growth literature assumes that researchers' labor is the only input into the R&D process (Romer, 1990; Jones, 1995, 1999; Ha and Howitt, 2007). Alternatively, some studies embrace the "lab equipment" specification of the R&D process, conditioning R&D output on the flow of R&D spending (Rivera-Batiz and Romer, 1991; Kruse-Andersen, 2017; Bloom et al., 2020).

In reality, though, both approaches may be limiting since it is likely that productivity in the R&D sector depends not just on the labor of researchers, but also on the services of *R&D capital*. R&D capital should be understood as a stock, accumulated over the years through targeted R&D investment. Indeed, anecdotal evidence suggests that R&D is an increasingly capital-intensive activity: new scientific ideas and technological blueprints increasingly rely on the effects of experimentation in sophisticated laboratories as well as on advanced numerical computation, rather than abstract philosophical reflection or pen-&-paper calculations. Modern R&D capital ranges from researchers' computing facilities to such extraordinary machinery as the Very Large Telescope (VLT) and the Large Hadron Collider.

Moreover, the practicality and complexity of research equipment has also undergone systematic, cumulative changes and productivity improvements over the decades and centuries. The difference for instance in usefulness of Ptolemy's astrolabe or Galileo's telescope, set against the VLT is breathtaking. Likewise, consider how early statisticians computed correlations and ran regressions without relying on computer processing capabilities. Modern R&D activity also increasingly uses AI algorithms, ranging from general-purpose tools like web search engines, to specific applications in genome sequencing or analysis of astronomical dataries, and sometimes even in solving long-standing problems, as in the case of DeepMind's AlphaFold which recently produced a major breakthrough in the protein folding problem.

Given this, how would the introduction of R&D capital affect our estimates and understanding of the economy's idea production function (IPF)? And what will be the implications of this

[1] Jones (2021) demonstrates that the Idea production function can be retrieved from either the Romer or 'quality-ladder' endogenous growth approaches.

extension for major questions such as whether ideas are getting harder to find, or whether the recent slowdown in total factor productivity (TFP) growth constitutes secular stagnation, or rather a temporary downswing? (e.g., Ramey, 2020).

In their influential paper Bloom et al. (2020) focus their attention around the following IPF,

$$\text{Research Output} = \underbrace{\text{Research Productivity}}_{\alpha_t} \times \text{Researchers}. \tag{1}$$

In other words, they postulate that research output, proxied by the rate of TFP growth in the economy, is proportional to the number of researchers. Since the latter rose dramatically over the post-war period whilst the former was fairly constant, this concentrates attention on how the "α" middle term, capturing the (potentially time varying) level of research productivity, has behaved. To achieve balance, the authors argue, α must have declined, indicating that research ideas have been getting harder to find. (See also our "A First Look at the Data" section below).

But would "idea TFP" still be strongly falling over time if the IPF also included R&D capital (in addition to R&D labor)? Consider the following log-linear (Cobb Douglas) specification where $\dot{A}_t/A_t$, TFP growth (theoretically representing the flow of new ideas), is a function of R&D labor (i.e., number of researchers, $\mathcal{R}$) but now also R&D capital ($\mathcal{K}$):

$$\frac{\dot{A}_t}{A_t} = \Gamma_t \mathcal{K}_t^{\beta} \mathcal{R}_t^{1-\beta}, \tag{2}$$

where $\beta \in [0,1]$ captures the share of R&D capital in the production of ideas, and Γ (like α above) captures unit research productivity. Predictions for idea TFP based on (1) will differ from those based on (2) if the rate of change in R&D capital systematically differs from that of R&D labor.[2] If both variables grow at a common rate, then the dynamics of both "idea TFP" concepts (α_t and Γ_t) will be the same. Otherwise, they will differ and idea TFP, as in (1), may in fact be systematically mis-measured, if not misleading.

However, although more general than (1), IPF (2) is *still* though quite restrictive: it implicitly imposes a unit elasticity of substitution between both R&D input factors and assumes factor productivity improvements behave in a neutral manner. In other words, if, say, R&D labor became relatively more expensive, on this basis firms could simply substitute 1-for-1 into R&D capital. Moreover, if R&D productivity changes over time (as it surely does) then the specification assumes that it impacts both R&D factors in the same manner. By contrast our empirical findings, on a less restrictive specification of the IPF (see equation (3) below), demonstrate that R&D capital is in fact

[2] Where $\mathcal{R}$ and $\mathcal{K}$ are some fraction of, respectively, the aggregate labor force and aggregate capital stock.

an *essential, complementary, and relatively scarce* factor in the R&D sector. In such a scenario, the relative scarcity of R&D capital will constrain R&D output even when (as is the case) R&D labor is abundant and fast growing. Notwithstanding, if that IPF specification (2) was correct, then using (1) instead would mechanically mis-attribute the observed discrepancy in growth rates between R&D labor and R&D output to falling idea TFP. Moreover, following standard omitted-variable bias reasoning, doing so would also attribute an incorrect weight to R&D labor (the β), depending on the true correlation between R&D factors.

Another important question is whether the growth rate of TFP is an appropriate measure of research output. TFP growth may reflect many other phenomena than just research output. For example, Baqaee and Farhi (2020) find that improvement in allocative efficiency, due to the reallocation over time of market shares to high-markup firms, accounted for about half of aggregate US TFP growth in 1997–2015. TFP measures may also conflate the cyclical volatility of capacity and factor utilization rates (Fernald, 2018), which are independent of technical progress.[3] In light of this, we opt instead for patent applications, a more direct measure of R&D output. An important implication of that choice is that patent applications, even relative to patents in force, have been growing over time in the US over the last decades while the TFP growth rate has declined. That in itself impacts on our estimates of idea TFP and makes the conclusion that "ideas are getting harder to find" less likely.

A First Look at the Data An initial glance at the US data (Table 1) suggests the following.[4] First, the number of new patent applications per researcher ($\Delta A/\mathcal{R}$) was gradually increasing over time. Maintaining exponential growth in idea production has indeed become more difficult, though: the ratio of new patent applications to patents in force grew slower than R&D employment. A similar conclusion is reached when using data on "effective R&D employment", defined by Bloom et al. (2020) as the ratio of total R&D expenditure to the average R&D wage, which grew slightly faster than raw R&D labor.

Second, "idea TFP" as defined in (1) depends crucially on the definition of research output. With patent applications as the output variable, the resultant measure of idea TFP is increasing over time. Declining idea TFP is only obtained once one identifies research output with patent applications relative to patents-in-force, for example like in Bloom et al. (2020) (cf. last line of

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[3] See also Figure A.21 in appendix A for the indexed profile of TFP. Whilst some countries such as the US and France have experienced a strong upward trajectory in TFP levels (albeit punctuated by low-growth episodes), other countries (e.g., Canada, Italy) have experienced trend breaks and decades-long stagnation of TFP. Taken at face this would suggest that those economies are in technical regress. Additional issues with TFP as a proxy of ideas are measurement issues, for example the provision of zero-price technologies.

[4] For more general discussion of recent US growth and productivity performance see Fernald and Wang (2016); Fernald et al. (2017) and for greater historical scope see Gordon (2016). See also Grossman et al. (2017) for links to income share developments.

Table 1: Summary Statistics of R&D Variables:
Average Annual Growth Rates (1968-2014)

Variables	Symbol	Growth Rate
Patent Applications	ΔA	3.211
Patents-in-Force	A	2.410
Patent Applications Relative to Patents-in-Force	$\Delta A/A$	0.782
R&D Capital	$\mathcal{K}$	3.394
R&D Labor	$\mathcal{R}$	2.099
R&D Wage	w	0.848
R&D Expenditure (Real)	Ω	3.319
R&D Expenditure Relative to R&D Wage	Ω/w	2.450
R&D Capital Relative to Patents-in-Force	$\mathcal{K}/A$	0.961
R&D Labor Relative to Patents-in-Force	$\mathcal{R}/A$	-0.304
Patent Applications Relative to R&D Labor	$\Delta A/\mathcal{R}$	1.090
Patent Applications Relative to Ω/w	$\Delta A/(\Omega/w)$	0.743
Patent Growth Relative to R&D Labor	$(\Delta A/A)/\mathcal{R}$	-1.289
Patent Growth Relative to Ω/w	$(\Delta A/A)/(\Omega/w)$	-1.628

Source: Derived from *WIPO, IPUMS CPS*.

Table 1).

Third, R&D capital grew almost exactly in line with growth in patent applications and noticeably faster than growth in R&D labor and the number of patents in force. This indicates that "idea TFP" growth measures which disregard the accumulation of R&D capital, such as (1), are most likely biased.

Contribution By introducing R&D capital alongside R&D labor into the IPF, and then estimating it allowing for a non-unitary elasticity of substitution and non-neutral unit productivity, our study fills an important gap in the empirical literature on R&D-based economic growth. We find that the elasticity of substitution between R&D capital and R&D labor in the IPF is about 0.6-0.8 and significantly below unity. This implies that R&D capital should be considered an essential factor in producing ideas, and complementary to R&D labor. We also identify a systematic positive trend in R&D labor productivity at about 1% per year on average and a cyclical dynamic in R&D capital productivity. On average, *effective* supply of R&D capital was lagging behind that of R&D labor, constraining R&D output. Idea TFP, the Hicks-neutral component backed out from the IPF, has not been falling but rather oscillating around a constant mean.

Accordingly, our results imply that ideas, instead of getting harder to find, in fact *require more sophisticated lab equipment* to be found. This is a scarcity which can only be bridged by increased accumulation and development of R&D capital, and not necessarily by employing more R&D staff. Because investments in R&D equipment are an endogenous variable that can be influenced by policy and institutions, our results contribute to lowering the assessment of the likelihood and inevitability of a secular stagnation in the future.

Organization Section 2 documents the construction of the time series of R&D capital as well as measurement of R&D labor and R&D output. We construct the stock of R&D capital in the post-war US economy, using the perpetual inventory method applied to BEA chain-type quantity indexes for R&D assets. Section 3 discusses the IPF and its estimation over 1968-2019, using a nonlinear system estimation technique with a flexible treatment of the unit productivity of R&D factors. Section 4 presents our results. We present several IPF forms, where R&D capital is included alongside R&D labor and where unit productivity in both R&D factors is modeled in an increasingly flexible manner. Section 5 takes our results and show how R&D can be decomposed over time into its constituent determinants; this illuminates which variables have or have not constrained the production of ideas. Thereafter, in Section 6 we derive idea TFP as the residual of the idea production function and comment on its properties. Section 7 concludes. Additional material is in the appendices.

2 Data and Measurement

We shall now discuss our empirical strategy of measuring capital, labor, and output in the R&D sector. A fundamental challenge here is to collect sufficiently long time series of acceptable proxies of the variables of interest. Since the available classification systems are not able to uniquely identify total R&D activity in the economy, we use a variety of auxiliary data sources that should provide conceptually close proxy variables for the concepts at hand. See Appendix A for a more extensive discussion of the data and transformations.

2.1 R&D Capital

To estimate R&D capital in the US economy we use Bureau of Economic Analysis (BEA) data. Unfortunately the BEA does not measure the aggregate R&D capital stock directly, nor does it publish long-run series on fixed-weights aggregates of R&D investment or R&D stock.[5] The reason for that is there are long-run trends in relative prices of inputs, such as the secular decline in prices

[5] The available data (in constant dollars) starts in 1999. This time span is however too short to analyze long-run patterns in R&D productivity.

of equipment relative to structures (Greenwood, Hercowitz and Krusell, 1997).

We construct the R&D capital stock using the perpetual inventory method. The capital stock is calculated as the sum of investment in period t and previous depreciated capital stock, $\mathcal{K}_t = (1-\delta)\,\mathcal{K}_{t-1} + I_t^{rd}$ where $\delta \in (0,1)$ is the depreciation rate of R&D capital and I_t^{rd} is real investment in R&D. This relationship is initialized in the standard manner: $\mathcal{K}_0 = I_0^{rd}/(g+\delta)$, where g is the long-run geometric growth rate of R&D investment. While the latter can be easily calculated from historical data, there is considerable uncertainty about the depreciation rate of R&D capital. We calibrate this rate at 15% per year. This number, which we understand as something of a consensus in the literature (Venturini, 2012), is much higher than that pertaining to the aggregate capital stock because of a relatively larger share of fast-depreciating equipment in R&D, and an accordingly lower share of structures.[6]

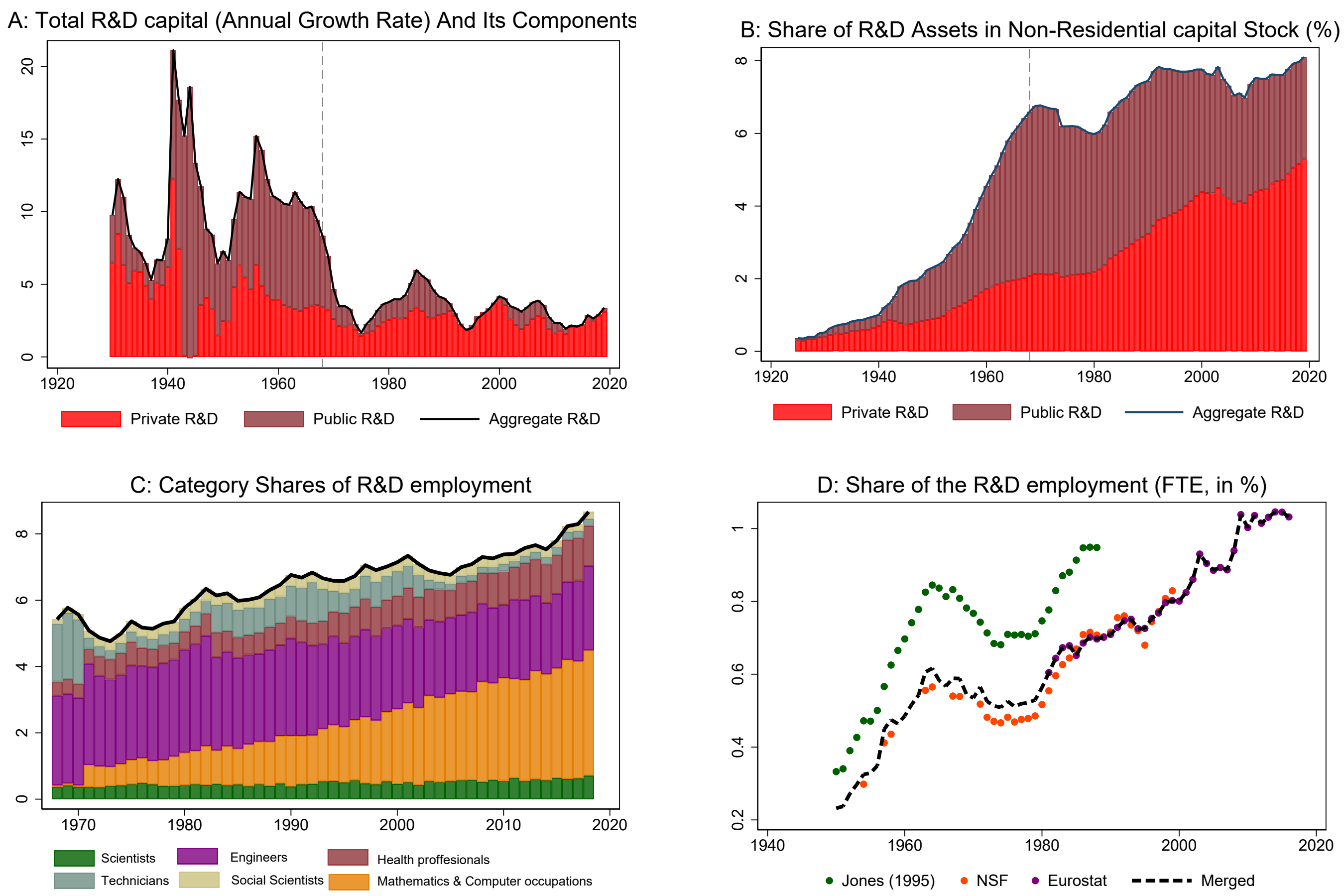
To obtain long-dated series of the total R&D capital and private R&D capital we proceed as follows. Since there are no measures of real R&D investment expressed in chained dollars we estimate it based on available series, i.e, nominal R&D investment data as well as price indexes. For the private sector, we divide nominal R&D investment (BEA code: `Y006RC`) by the price index of this asset (`Y006RG`). The same strategy is applied for the public sector (`Y057RC` and `Y057RG`, respectively). In addition, we also consider the following components of public investment: Federal Non-Defense (`Y069RC` and `Y069RG`), Defense (`Y076RC` and `Y076RG`) and state and local (`Y073RC` and `Y073RG`). R&D capital stock in the US since 1929 (under the baseline calibration) is plotted in Figure 1, panel A. In turn, the R&D capital share in the total nonresidential capital stock is shown in panel B.

2.2 R&D Labor

The second factor in the IPF is R&D labor. At the conceptual level – and in line with the definition from the Frascati Manual (OECD, 2015) – this category refers to all employees who undertake creative work aimed at general increases in the existing stock of knowledge. In practice, however, application of this definition requires very detailed information about tasks that are related to R&D activities. According to the best of our knowledge such data are not available, making it effectively impossible to measure R&D labor directly.

[6] Several recent studies provide empirical evidence that suggests that the depreciation rate of R&D capital may be even higher than 15%. Bernstein and Mamuneas (2006) find that this deprecation rate is above 15% while Li and Hall (2019) place it even above 30%. In the recent KLEMs (2019) edition, the depreciation rate for R&D assets is fixed at 20%.[7] In addition, the BEA publish historical series on depreciation of R&D assets in the US and, according to the BEA estimates, the implied depreciation rate is slightly above the consensual value of 15%. Appendix A discusses the sensitivity of the constructed series to different starting values, depreciation rates and different data sources.

Figure 1: R&D Capital and R&D Labor



Notes: The gray vertical dashed line in panels A and B indicates the start of the estimation sample.

To circumvent this, we use two strategies of indirect measurement. First, we estimate the labor input in R&D activity using micro-data which contains information about the structure of occupations. An ideal strategy would be to use detailed data on skills/abilities content in occupations and merge them with the occupational structure of the labor force. The most important caveat to this approach, however, is that the task-content of jobs changes over time and to the best of our knowledge, there is no longitudinal survey on research intensity across occupations. For instance, the O*NET database offers estimates on a range of skill and abilities intensities of occupations, but there is no direct measure of research intensity and the time span of this dataset is quite short (the survey started in 1998).

Thus instead we use IPUMS CPS data (Ruggles et al., 2019).[8] This database offers harmonized micro data from the Current Population Survey (CPS), i.e., the monthly U.S. labor force survey. Based on the conceptual definition of R&D personnel and scientists and engineers (S&E) we can identify the following occupational groups whose work could be classified as embodying R&D activity[9]:

> **Scientists** Agricultural and Food Scientists (*IPUMS code* 1600); Biological Scientists (1610); Conservation Scientists and Foresters (1640); Medical Scientists, and Life Scientists, All Other (1650); Astronomers and Physicists (1700); Atmospheric and Space Scientists (1710); Chemists and Materials Scientists (1720); Environmental Scientists and Geoscientists (1740); Physical Scientists, nec (1760).
>
> **Mathematical & Computer Occupations** Actuaries (1200); Operations Research Analysts (1220); Statisticians (1230); Mathematical science occupations, nec (1240); Computer Scientists and Systems Analysts/Network systems Analysts/Web Developers (1000); Computer Programmers (1010); Software Developers, Applications and Systems Software (1020); Computer Support Specialists (1050); Database Administrators (1060); Network and Computer Systems Administrators (1100).
>
> **Engineers** Architects, Except Naval (1300); Surveyors, Cartographers, and Photogrammetrists (1310); Aerospace Engineers (1320); Chemical Engineers (1350); Civil Engineers (1360); Computer Hardware Engineers (1400); Electrical and Electronics Engineers (1410); Environmental Engineers (1420); Industrial Engineers, including Health and Safety (1430); Marine Engineers and Naval Architects (1440); Materials Engineers (1450); Mechanical Engineers (1460); Petroleum, mining and geological engineers, including mining safety engineers (1520); Engineers, nec (1530); Drafters (1540); Engineering Technicians, Except Drafters (1550); Surveying and Mapping Technicians (1560).
>
> **Technicians** Agricultural and Food Science Technicians (1900); Biological Technicians (1910); Chemical Technicians (1920); Geological and Petroleum Technicians, and Nuclear Technicians (1930); Life, Physical, and Social Science Technicians, nec (1960); Professional, Research, or Technical Workers, nec (1980).

[8] See also https://cps.ipums.org/cps.

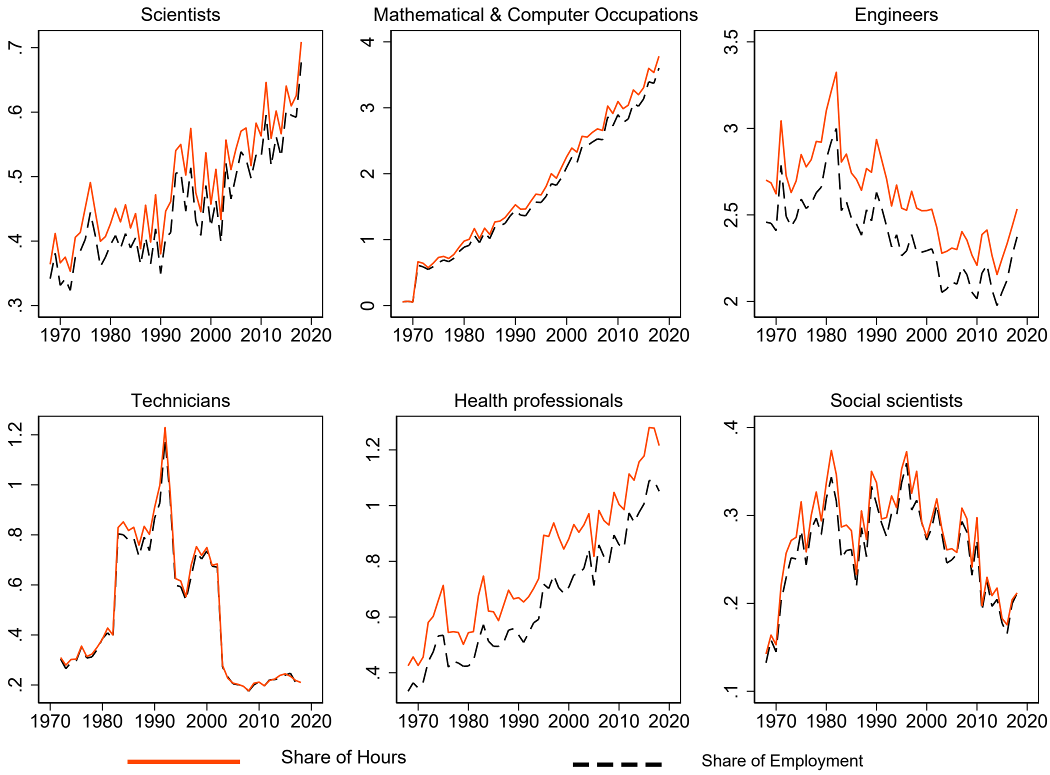
[9] In practice, we try to match the Eurostat definition of human resources in science and technology. According to this definition, scientists engineers (S&E) are workers who conduct research, improve or develop concepts, theories and operational methods and/or apply scientific knowledge relating to their fields. This definition can be covered by following groups of occupations (according to the ISCO-08 classification): Science and engineering professionals (21), Health professionals (22) and Information and communications technology professionals (25).

Social Scientists Economists and market researchers (1800); Psychologists (1820); Urban and Regional Planners (1830); Social Scientists, nec (1840).

Health Professionals Chiropractors (3000); Dentists (3010); Dietitians and Nutritionists (3030); Optometrists (3040); Pharmacists (3050); Physicians and Surgeons (3060).

The relative shares of these groups over time can be seen in Figure 1, Panel C. And for the same group, Figure 2 calculates aggregate hours worked in the US economy.

FIGURE 2: SHARE OF R&D RELATED OCCUPATION GROUPS IN TOTAL US EMPLOYMENT AND HOURS WORKED



Hence, in our baseline definition R&D labor includes scientists, Mathematical & Computer occupations, Engineers, Technicians, Social Scientists and Health professionals. For such a group, we calculate aggregate hours worked in the US economy. As an alternative operational definition of R&D labor (and a robustness check) we also narrow the set of occupations to scientists, Mathematical & Computer occupations and Engineers (Figure 1, Panel C shows the evolution of such occupational groups over time), in terms of employment and hours worked shares).

Our second strategy of indirect measurement is the following. For the sake of a further robustness check we also take advantage of publicly available data on R&D employment and merge historical series. We begin with official estimates of R&D activity published by statistical offices, Eurostat and the OECD. Unfortunately the Eurostat/OECD series begins only in 1981. To overcome this we use older data vintages to extrapolate the existing series. For the time period 1968-1980 we use data collected within the IRIS (Industrial Research and Development Information System) programme conducted by the NSF (National Science Foundation). Moreover, based on historical data from Jones (1995) it is possible to further extrapolate the observations backwards, i.e., into the 1950s-60s (see Figure 1, Panel D.)

2.3 R&D Output

The choice of the output variable in the IPF is also challenging. As one possible approach, since R&D encompasses activities that are aimed at reducing unit costs of production or increasing the variety of goods offered, one could measure the aggregate stock of knowledge/technology as the level of TFP in the economy. In turn, the flow of R&D output would be represented as increases in aggregate TFP over time.

However, although popular, the strategy of using TFP growth as a proxy of R&D output is problematic.[10] First, changes in TFP might be driven by changes in technology but they may also could result from other processes. For instance, reduction in mis-allocation could increase measured TFP (Oberfield and Raval, 2021). Other potential causes include, e.g., production function mis-specification or changes in the internal composition of production factors. Second, measured TFP sometimes falls over time while the functional form of the IPF requires positive values of R&D output. This condition makes increases in TFP a less applicable proxy since there have indeed been historical periods of decline in the TFP in the US.[11]

Another strategy in measuring the aggregate macroeconomic outcome of the R&D sector is to use patent data. A common practice in the related literature is to use *patent applications* as the R&D output variable (Madsen, 2008; Ang and Madsen, 2011; Venturini, 2012). This is the approach we take. Since we are interested in long historical patent data our principal measures are taken from Marco et al. (2015) which are updated with the recent WIPO (World Intellectual Property Organization) series.

2.4 R&D Rental Prices

Finally, identification of the elasticity of substitution between R&D factors and the nature of unit productivities requires data on relative rental prices. We calculate the capital rental rate as the sum of the real interest rate and the R&D capital deprecation rate. Specifically we use the interest

[10] There is also an open discussion as to whether one should use absolute or relative increases in TFP ($\dot{A}$ or $\dot{A}/A$, respectively) as the flow concept of R&D output (see Bloom et al., 2020).

[11] In the associated literature, the problem of negative TFP growth in estimating IPFs is overcome by considering an approximation of the IPF (Ha and Howitt, 2007) or by taking 5-year averages (Ang and Madsen, 2011).

rate on 10-year government bonds (FRED code: GS10) deflated by the GDP deflator (GDPDEF).[12]

For the rental price of labor, we calculate the real hourly wage. The CPS dataset contains sufficient information about wages, allowing us to construct long-dated series on real wages for our baseline measure of R&D labor. It also enables us to construct the series of real hourly wages for the alternative measure that uses a narrower set of occupations. In the case of the merged historical series on R&D labor we use the same real wages as in our baseline since there is no publicly available long series on wages in the R&D sector.

3 Idea Production Function

3.1 Constant Elasticity Specification

Following our earlier discussion, we estimate the following IPF:

$$\Delta\widetilde{A}_t = \left[\eta\left(\Gamma_t^{\mathcal{K}}\widetilde{\mathcal{K}}_t\right)^{\frac{\xi-1}{\xi}} + (1-\eta)\left(\Gamma_t^{\mathcal{R}}\widetilde{\mathcal{R}}_t\right)^{\frac{\xi-1}{\xi}}\right]^{\frac{\xi}{\xi-1}} \tag{3}$$

where ΔA_t is the flow of new ideas (as represented by new patent applications). The IPF is written in 'normalized' (or indexed) form. Thus, $\widetilde{X}_t = X_t/X_z$ where $X_z > 0$ denotes the value of X at the point of normalization.[13] Distribution parameter $\eta \in [0,1]$ measures the steady-state level of the R&D capital share in total R&D income. Parameter $\xi > 0$ is the elasticity of substitution between R&D capital and R&D labor, with the special cases of Leontief, log-linear and linear forms, respectively, given by $\xi \to 0, 1, \infty$.[14] IPF form (3) relaxes the assumption of a unit elasticity of substitution and accommodates the possibility of factor-specific productivity improvements over time, whose paths are captured by $\Gamma_t^{\mathcal{K}}$ and $\Gamma_t^{\mathcal{R}}$ for R&D capital and labor, respectively.

It is well-known that estimation of production relationships is improved by joint estimation with the first order conditions (FOC), León-Ledesma, McAdam and Willman (2010). This is because such an approach combines information from different sides of the production framework (costs and volumes) and exploits cross-equation restrictions. In the case of the considered IPF

[12] We have also experimented with CPI as the price proxy; results remain almost unchanged.

[13] Without explicit normalization, parameter estimates in constant-elasticity functions can be shown to be scale dependent, arbitrary and un-robust. Normalization points are averages. For linear series such as a time trend, they are given by the arithmetic mean; otherwise geometric averages are used. See León-Ledesma, McAdam and Willman (2010) for Monte-Carlo analyses, and La Grandville (1989) and Klump and de La Grandville (2000) for the seminal theoretical contributions.

[14] Thus (2) emerges as a special (and testable) case: $\xi = 1$; and $\Gamma_t^{\mathcal{K}} = \Gamma_t^{\mathcal{R}}$.

and, after taking logs and combing the FOCs, this implies the proportionality:

$$\ln\left(\frac{r_t^{\mathcal{K}}\mathcal{K}_t}{w_t^{\mathcal{R}}\mathcal{R}_t}\right) = \left(\frac{\xi-1}{\xi}\right)\ln\left(\frac{\Gamma_t^{\mathcal{K}}\widetilde{\mathcal{K}}_t}{\Gamma_t^{\mathcal{R}}\widetilde{\mathcal{R}}_t}\right) \tag{4}$$

where $r_t^{\mathcal{K}}$ is the real rental price of R&D capital, $w_t^{\mathcal{R}}$ denotes real wages in the R&D sector.[15] Thus, estimation consists in the joint system estimation of parameters in the first-order condition (4) with IPF (3) (in the latter case, we also transform the specification into logs).

3.2 Specification of Unit Productivity Terms

Another important decision to make relates to the assumption about the trajectory of productivity improvements to the R&D factor inputs over time. The latent nature of both processes (and the Diamond–McFadden impossibility theorem in standard production theory) requires that assumptions are made about each of them.

We consider three increasingly more sophisticated assumptions about the growth in unit productivity of R&D factors in the IPF. The first two are nested in the Box–Cox form $\log \Gamma_t^j = \mathbf{B}(\gamma_j, \lambda_j; t)$. The log-level of productivity to each j R&D factor is increasing around a normalized or average growth rate γ_j, where parameter $\lambda_j \in \mathbb{R}$ determines shape. If $\lambda_j = 1$ then the level of unit productivity increases linearly over time at a constant growth rate γ_j. Otherwise the growth is accelerating ($\lambda_j > 1$) or decelerating ($\lambda_j < 1$) relative to the mean γ_j (see Appendix B for more details on the Box–Cox form).

The third case is where we consider a functional form that allows us to account for the possibility of unknown structural breaks (or long swings). This is a good robustness check in itself but is also motivated by some evidence of structural instability in the patent growth process (see online Appendix C). In that case we use a Fourier expansion: $\log \Gamma_t^j = \mathbf{F}\big(\gamma_j, \kappa_j^{sin}, \kappa_j^{cos}; t\big)$.[16] Any possible structural breaks or cycles around its trend growth rate of γ_j will thus be captured by the κ parameters.

Due to substantial variation and a possible appearance of structural breaks, the normalization

[15] Estimating the capital and researcher R&D FOCs separately can be problematic. Accordingly, among these three equations (i.e., two first order conditions plus their ratio), we use the relative factor share equation (4), for the following reasons. First, the share equation contains information on both forms of factor productivities over time, rather than just one. Second, it does not require any information about the dynamics of markups. The individual FOCS are based on the assumption of perfect competition. The share equation remains useful if markups are positive but stable over time. However, recent empirical literature has documented a secular upward trend in markups in the US (De Loecker, Eeckhout and Unger, 2020), albeit aggregate (rather than R&D-specific) ones. This could potentially lead to a systematic bias in the estimation of the individual FOCs. At the same time, in the first-order condition using the relative factor share (4) markups are eliminated. Third, condition (4) does not require any information about the prices of new ideas.

[16] See Christopoulos and León-Ledesma (2010) for a discussion of Fourier forms in economics. We follow Ludlow and Enders (2000) who showed that a single frequency is invariably sufficient to approximate the Fourier expansion in the bulk of empirical applications.

point for R&D factor shares seems far from obvious. We start with setting the distribution parameter η at $1/3$ (typical of the long-run average of the total capital income share). Making this assumption reflects the fact that there is no reliable data that allows us to estimate factor shares in the R&D sector. However, we also include cases wherein η is estimated. Finally, we estimate using a nonlinear system estimators, which takes into account cross-equation correlation of the residuals, as well as accounting for the cross-equation parameter constraints.

4 Results

4.1 Baseline Results

The first section of Table 2 presents the various parameter estimates and the expression of the R&D productivity terms ($\mathbf{B}$ and $\mathbf{F}$ denotes the Box–Cox and Fourier forms respectively, and "Exp." denotes exponential). The middle section presents tests of relevant parameter restrictions, and the final section shows estimation diagnostics. The first two rows in that final section refer to ADF test of the unit root null associated to the errors in equations (4) and the logged form of (3); p-values are obtained by bootstrapping. Finally terms ll, bic and $rmse$ denote, respectively, the Log Likelihood, and the Bayesian Information Criterion, and the Root Mean Square Error.

Case 1 estimates an IPF with only R&D labor (akin to equation (1)), followed in cases 2 and 3 by the IPF augmented with R&D capital (equation (2)), without and without the unit-elasticity constraint. All forms produce superficially not entirely unreasonable results: the first yields a power coefficient of 1.43, the second and third imply a growth rate of (Hicks neutral) R&D unit productivity of around $1.2 - 1.3\%$ per year (close to the R&D labor rates in subsequent specifications). However the diagnostics suggest a poor fit to the data; or at least that these case are dominated by the additional cases.

Cases 4 and 6 introduce the Box-Cox unit productivity forms, first for R&D labor then for both R&D input factors, whilst case 5 imposes simple exponential productivity growth for both factors. The case for a unitary substitution across these cases is mixed: case 6 illustrates the severe and well-known issue of identifying productivity terms when $\xi \approx 1$ (Sato, 1970); column 4 produces an unusually high elasticity value. All three cases suffer diagnostic issues, for instance the residuals exhibit extreme persistence and non stationarity.

Table 2: Baseline Results

Parameter, Case	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
β	1.428^{***}							
	(0.069)							
ξ		1.000	0.844^{***}	2.531^{***}	0.737^{***}	0.986^{***}	0.793^{***}	0.760^{***}
		$(-)$	(0.169)	(0.920)	(0.139)	(0.169)	(0.019)	(0.062)
$\gamma_{\mathcal{R}}$		0.013^{***}	0.012^{***}	0.012^{***}	0.011^{***}	-0.013	0.011^{***}	0.011^{***}
		(0.002)	(0.002)	(0.003)	(0.001)	(0.308)	(0.001)	(0.001)
$\lambda_{\mathcal{R}}$				2.890^{***}		6.453		
				(0.766)		(19.078)		
$\gamma_{\mathcal{K}}$					0.004	0.060	-0.016^{***}	-0.013^{***}
					(0.004)	(0.615)	(0.003)	(0.004)
$\lambda_{\mathcal{R}}$						5.208		
						(3.885)		
$\kappa_{\mathcal{K}}^{sin}$							0.556^{***}	0.438^{***}
							(0.045)	(0.137)
$\kappa_{\mathcal{K}}^{cos}$							-0.427^{***}	-0.337^{***}
							(0.028)	(0.109)
η								0.418^{***}
								(0.121)
R&D Labor Productivity	no	Exp.	Exp.	**B**	Exp.	**B**	Exp.	Exp.
R&D Capital Productivity	no	Exp.	no	no	Exp.	**B**	**F**	**F**
η	fixed	fixed	fixed	fixed	fixed	fixed	fixed	estimated
$\xi = 1$			[0.357]	[0.096]	[0.058]	[0.934]	[0.000]	[0.000]
$\lambda_{\mathcal{R}} = 1$				[0.014]		[0.775]		
$\lambda_{\mathcal{K}} = 1$						[0.279]		
$\gamma_{\mathcal{R}} = \gamma_{\mathcal{K}}$					[0.086]	[0.937]	[0.000]	[0.000]
$\kappa_{cos}^{\mathcal{K}} = \kappa_{sin}^{\mathcal{K}} = 0$							[0.000]	[0.006]
res_4		[0.262]	[0.086]	[0.066]	[0.101]	[0.020]	[0.006]	[0.008]
res_3	[0.413]	[0.531]	[0.095]	[0.237]	[0.085]	[0.051]	[0.001]	[0.000]
ll	16.5	81.1	78.7	96.9	76.5	100.7	133.2	134.2
bic	-29.0	-150.4	-141.7	-174.2	-133.3	-173.9	-239.0	-237.0
$rmse_4$		0.140	0.140	0.124	0.137	0.129	0.089	0.089
$rmse_3$	0.177	0.139	0.138	0.116	0.138	0.097	0.049	0.049

Notes: The numbers in parentheses are robust standard errors, where the significance stars are to be read as $^{*} < 0.1,^{**} < 0.05,^{***} < 0.01$. Probability values are in brackets. Symbols $\mathbf{B}$ and $\mathbf{F}$ denotes the Box-Cox and Fourier forms respectively, and "Exp." denotes exponential. Cases 2 and 3 (the Cobb Douglas cases) assume Hicks neutrality. In the diagnostic section of the table, the first two rows refer to ADF test of the unit root null associated to the errors in equations (4) and the logged form of (3) and the p-values are obtained by bootstrapping distribution. Thereafter we have Wald tests of various parameter restrictions. Finally terms ll, bic and $rmse$ denote, respectively, the Log Likelihood, and the Bayesian Information Criterion, and the Root Mean Square Error.

The final two cases are the most data congruent (witness the dramatic improvement in diagnostic measures). We impose constant growth in R&D labor productivity (consistent with the results of column 6) and allow R&D capital productivity to follow the Fourier form (the difference between cases 7 and 8 is that the latter freely estimates the distribution parameter[17]). Both Fourier parameters are statistically significant, and of opposite signs implying a somewhat cyclical trajectory for R&D capital productivity (the point estimates of the normalized productivity growth of R&D capital are negative, but this is precisely an average over a cyclical trajectory). Indeed, the role of structural breaks and swings is actually predominant over the sample (see also next section) such that there is no visible downward trend in R&D capital augmentation. R&D labor productivity is increasing by 1.1% per year. In contrast to previous estimates, non-stationarity in residuals can be decisively rejected. The substitution elasticity, finally, is significantly below unity (around $0.7 - 0.8$). Thus, R&D capital and R&D labor are gross complements in the IPF. Unit labor productivity is increasing in the R&D sector, while unit capital productivity exhibits strong non-linear variability.

4.2 Robustness

As a robustness check we consider alternative empirical measures of R&D capital and R&D labor. The additional estimates for fixed and estimated η are presented in Table D.1. All these results replicate our previous preferred findings: the elasticity of substitution ξ is below unity; the average growth rate of R&D labor productivity is ranging from $0.1\% - 2.6\%$ per annum; there is thus evidence in favor of the presence of a cyclical dynamic / multiple structural breaks in R&D capital productivity.

5 Decomposition of Ideas Growth

An instructive exercise is to use our preferred estimates to decompose the sources of ideas production into its constituent elements: R&D factors and R&D productivity. This can illuminate which elements constrain or encourage ideas production over time.

Specifically, using the IPF (3) we decompose growth in new patent applications as follows:

$$g_{\triangle \widetilde{A}_t} = \Pi_{K,t}(g_{\Gamma_t^{\mathcal{K}}} + g_{\widetilde{\mathcal{K}}_t}) + \Pi_{R,t}(g_{\Gamma_t^{\mathcal{R}}} + g_{\widetilde{\mathcal{R}}_t}) \tag{5}$$

[17] Parameter values did not prove very sensitive to different imposed η values. We also tried a number of different non-linear system estimation methods but with little variation in estimates or inference. Details available.

where

$$\Pi_{K,t} = \eta \left(\frac{\Gamma_t^{\mathcal{K}} \widetilde{\mathcal{K}}_t}{\Delta \widetilde{A}_t} \right)^{\frac{\xi - 1}{\xi}}, \tag{6}$$

$$\Pi_{R,t} = (1 - \eta) \left(\frac{\Gamma_t^{\mathcal{R}} \widetilde{\mathcal{R}}_t}{\Delta \widetilde{A}_t} \right)^{\frac{\xi - 1}{\xi}} \tag{7}$$

are the respective factor shares in R&D. We use the theoretical values for new patent applications, i.e. the values explained by IPF, excluding regression residuals. Furthermore, aiming to capture secular trends in ideas production rather than high-frequency fluctuations, data on R&D inputs and output have been HP-filtered prior to the decomposition (using $\lambda = 6.25$, annual data).

FIGURE 3: THE IDEA GROWTH DECOMPOSITION
(ANNUAL CHANGE ON HP-FILTERED CONTRIBUTIONS)

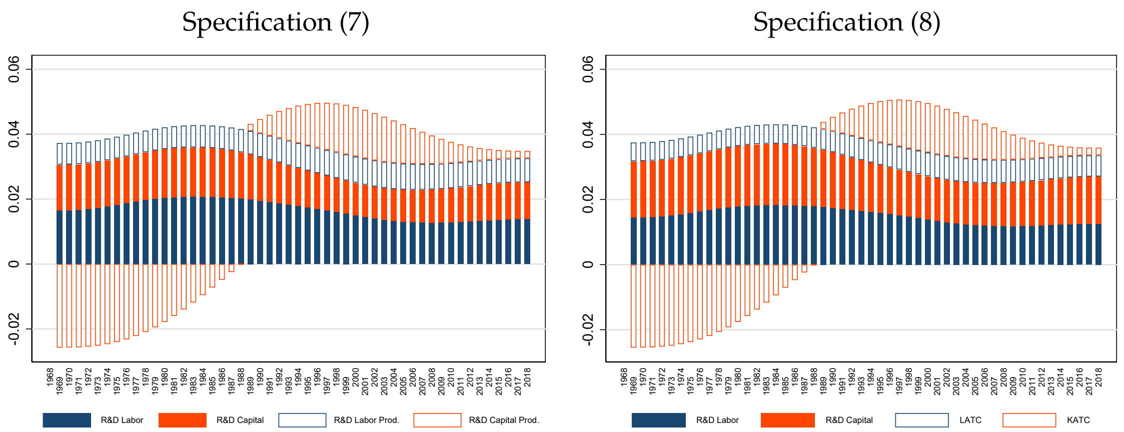


Figure 3 decomposes the growth of ideas into factor and productivity terms, using the last two specifications in Table 2. From this we can perhaps identify three main phases of ideas growth: (i) sluggish growth in ideas (up to early 1980s), (ii) sharp acceleration in ideas growth (1980s-2000s), and (iii) slowdown in ideas growth (since 2000s). The relative contribution of R&D capital vs. R&D labor depends on the model specification – either they are roughly equal or R&D labor is somewhat more important – but in any case the time trends of both contributions are largely parallel to one another, namely both are relatively steady over time, with just a minor increase around the early 1980s and a minor decrease in late 2000s. Furthermore, as labor productivity is growing uniformly at $\sim 1\%$ per year, its contribution to ideas production is also steady, and quantitatively somewhat less important than the contribution of input growth. The three phases of ideas growth are accounted for exclusively in the strong cyclical dynamic of R&D capital productivity. The contribution of that factor to ideas growth was strongly negative in phase (i), then sharply increased into the positive domain in phase (ii), and then gradually fell back to about zero in phase (iii).

Interestingly, the timing of the three phases coincides with the adoption of ICTs as major general purpose technologies in the 1980s-2000s (Bresnahan and Trajtenberg, 1995; Jorgenson, 2005; Aum, Lee and Shin, 2018). Universities and research labs in the US were among the earliest adopters of both technologies. In that same period, R&D capital productivity increased markedly. In turn, according to our results the episode of R&D capital productivity growth ended around the time of the global financial crisis. It is conceivable that it will resume one day in the future, perhaps after a breakthrough in artificial intelligence (Brynjolfsson, Rock and Syverson, 2019; Growiec, 2022).

6 Is Idea TFP Falling Over Time?

With a constant elasticity IPF specification, there is no unique idea TFP. Instead, unit productivity of each factor $(\Gamma_t^{\mathcal{K}}, \Gamma_t^{\mathcal{R}})$ is separately identified. With this in mind, however, one can nevertheless calculate a joint "idea TFP" factor capturing Hicks-neutral technical change in R&D.[18] Specifically, we calculate the log of idea TFP from the IPF (3) as follows:

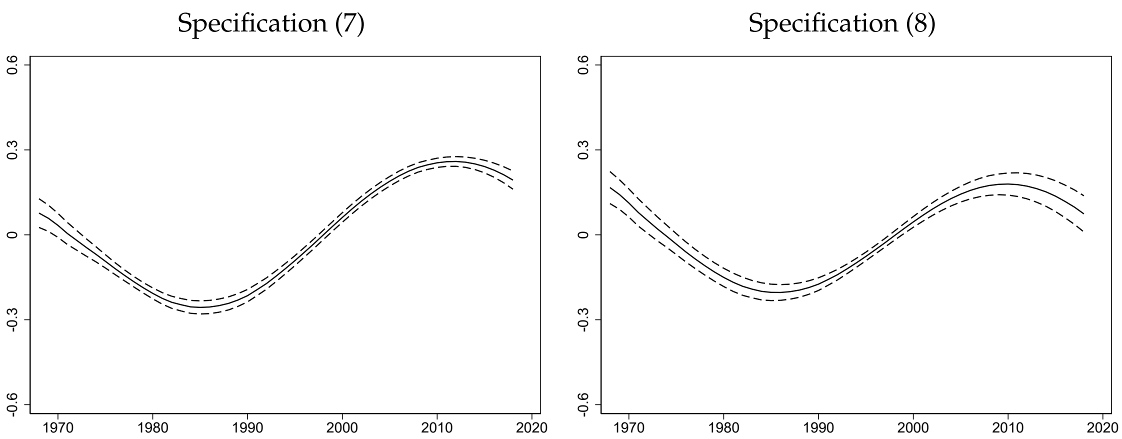
$$\log(\widetilde{TFP}) = \frac{\xi}{\xi - 1} \log \left[\frac{\eta \left(\Gamma_t^{\mathcal{K}} \widetilde{\mathcal{K}}_t\right)^{\frac{\xi-1}{\xi}} + (1-\eta)\left(\Gamma_t^{\mathcal{R}} \widetilde{\mathcal{R}}_t\right)^{\frac{\xi-1}{\xi}}}{\eta \left(\widetilde{\mathcal{K}}_t\right)^{\frac{\xi-1}{\xi}} + (1-\eta)\left(\widetilde{\mathcal{R}}_t\right)^{\frac{\xi-1}{\xi}}} \right]. \tag{8}$$

The results are plotted in Figure 4. In contrast to Bloom et al. (2020) we do not find a sharp drop in idea TFP, rather a wave oscillating around a constant mean. Along with the three phases in ideas growth, identified in Section 5, idea TFP first falls (until 1980s), then grows (from 1980s up to about 2010), and then begins to fall again.

We interpret our results as an indication that R&D capital is an essential, complementary factor in R&D activity. In R&D, like in the aggregate economy, capital accumulation markedly outruns the growth in labor supply over the long run. In effective terms, though, factoring in the systematic increases in R&D labor productivity and much more erratic behavior of R&D capital productivity over the period 1968-2019, average growth in R&D labor outran that of R&D capital. On top of this trend, the effective R&D capital-to-labor ratio also exhibited a clear cyclical pattern, following the three main phases of ideas growth which we identified above (Figure 5). This may indicate

[18] The mapping from the pair $(\Gamma_t^{\mathcal{K}}, \Gamma_t^{\mathcal{R}})$ to Hicks-neutral idea TFP is not invertible. There is a second dimension of technical change, absent in the concept of idea TFP: *factor bias* in technical change (see Klump, McAdam and Willman, 2012).

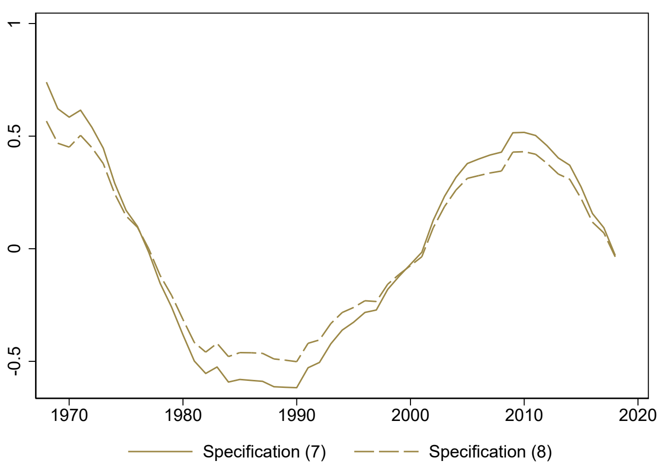
Figure 4: Idea TFP backed out from the IPF



Notes: Dashed lines represent confidence intervals. Idea TFP retrieved from the last columns in table 2 presented in logs.

that Bloom et al. (2020) celebrated result that "ideas are getting harder to find" should be in fact reinterpreted as "there is an increasing scarcity of R&D capital required to find the new ideas", with a policy implication that R&D output could be increased by subsidizing and facilitating the accumulation of state-of-the-art R&D capital rather than necessarily increasing R&D employment.

Figure 5: Effective R&D Capital-to-Labor Ratio (Log)



In relation to the debate whether the observed slowdown in TFP growth over the last decades is a sign of an upcoming secular stagnation (Jones, 2002; Gordon, 2016) or represents a transition phase to a digitally mature economy which would again grow faster once the transition period is over (Brynjolfsson and McAfee, 2014; Brynjolfsson, Rock and Syverson, 2019), our results are indicative of the latter option. According to our estimates, the current slowdown in R&D output is likely due to a relative shortage of R&D capital, rather than sharply falling idea TFP.[19]

7 Conclusion

We introduced R&D capital alongside R&D labor into the Idea Production Function, and estimated it using a flexible, non-neutral, non-constant elasticity specification. We find that the elasticity of substitution between R&D inputs in the IPF is $0.7-0.8$ and significantly below unity. This implies that R&D capital should be considered an essential factor in producing ideas, and complementary to R&D labor – in other words the marginal productivity of R&D labor will be enhanced by the presence of R&D capital.

We also identify a systematic positive trend in R&D labor productivity at about 1% per year on average and a cyclical trend in R&D capital productivity. Our results suggest that, cyclical variability aside, the effective supply of R&D capital was systematically lagging behind R&D labor, constraining R&D output over the long run.

Our results imply that ideas, rather than simply getting harder to find, in fact *require more sophisticated lab equipment* to be found (or implemented). This is a scarcity which can only be bridged by increased accumulation and development of R&D capital, not necessarily just by employing more R&D staff. Moreover, because investments in R&D equipment are an endogenous variable that can be influenced by policy, our results suggest a weakened case for future secular stagnation.

Our analysis could be extended in a number of dimensions. First, one could use international panel data or aggregated global-level data on R&D inputs and output to gauge whether our results hold more broadly. Second, one could consider alternative measures of R&D output. Using TFP growth is a usual alternative, albeit plagued with measurement issues and the necessity to make potentially arbitrary decisions such as, for example, applying a smoothing filter to accommodate the randomness in the timing of adoption of new technologies and the presence of episodes of falling TFP levels due to reasons unrelated to the actions of the R&D sector. Finally, our results could be used in theoretical studies aiming at understanding the mechanisms of long-run growth in presence of R&D capital. In Growiec (2019, 2022) we take some first steps in that direction.

[19] Speculatively, the most promising kind of R&D capital required to achieve progress is probably AI algorithms.

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A Data Construction

A fundamental problem in the current research project is a collection of the long time series that would be acceptable proxies of variable of interest. We will discuss in details specific problems, e.g. long-run trends on relative prices or changes on occupational systems, that affects precision of our measurement strategy.

A.1 Output

From an economic perspective, R&D activities aim to reduce unit cost of production or increase the variety of offered goods. At the aggregate level, the existing stock of knowledge/technology can be presented by the total factor productivity (TFP). Specifically one can use TFP estimates provided by Fernald (2018) which account for changing capacity utilization. Since the TFP is measured residually adjustment by capacity utilization reduces unwarranted variation due to changes in short-run factors, i.e., demand fluctuation. Second, one could use the Multifactor Productivity (MFP) index provided by OECD. Thirdly, one could use the latest Penn World Table estimates of the TFP (Feenstra, Inklaar and Timmer, 2015).

However, the using TFP as a proxy of the R&D output yields some problems. First, any changes in TFP might be driven by changes in the technology but also it could result from other processes. For instance, reduction in miss-allocation could improve TFP stock. Second, the TFP is the stock variable and, as a result, the output of the R&D sector is related to changes in existing technology so it could be measured by growth rates of the TFP. At the same time, functional form of the Idea production function requires positive values of the output. This condition makes the TFP growth less applicable proxy as there could be some periods/events of decline in the TFP.[1]

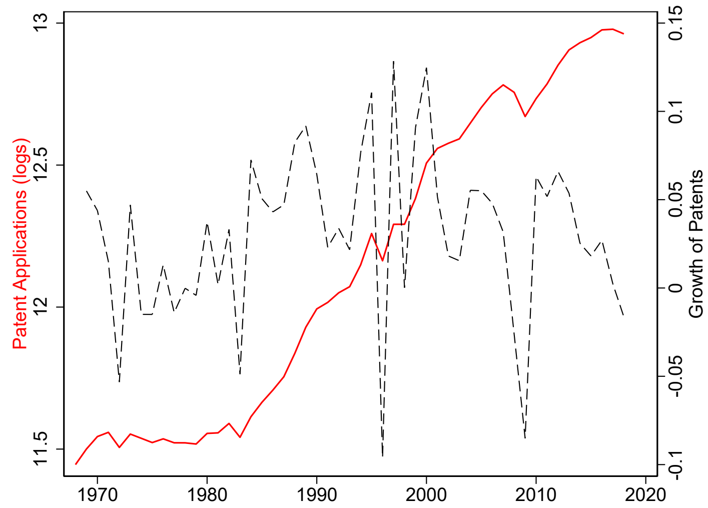
Another strategy in measuring an aggregate macroeconomic outcome of the R&D sector is to use patent data. A common practice in related literature is to use the patent applications (Madsen, 2008; Ang and Madsen, 2011; Venturini, 2012). Since we are interested in long historical patent data our principal measure is taken from Marco et al. (2015). The series of interest – the flow of new patent applications – is portrayed on figure A.1.

A.2 R&D Capital

We use the Bureau of Economic Analysis (BEA) data to estimate R&D capital in the US economy. The key problem in measuring the total R&D capital stock is a fact that there is no available

[1] In the associated literature, the problem of negative TFP growth in estimating idea production function is overcome by considering approximation of idea production function (Ha and Howitt, 2007) or by taking 5-year averages (Ang and Madsen, 2011).

Figure A.1: New patent applications in the US, 1968-2019



Notes: In this figure we plot on the lhs axis the log of patent applications and on the rhs the growth rate of patents.

aggregate R&D capital stock, i.e., combining private and public sector. At the same time, there the BEA does not publish long-run series on fixed-weights aggregates of R&D investment or R&D stock.[2] The reason for that is that are long-run trends in the relative prices of inputs, long-run decline in relative prices of investment (Greenwood, Hercowitz and Krusell, 1997). Therefore, we consider two approaches. First, we use direct measure of fixed assets. Second, the standard perpetual inventory method (PIM) is applied to the investment series in order to estimate the R&D capital.

In our first approach, we aggregate the available series on the R&D fixed assets. Although the BEA does not provide data on aggregate R&D assets it offers detailed series private and government sector. In particular, we use the BEA chain-type quantity indexes for R&D assets in both private (BEA code: `kcntotl1rd00`) and public sector (BEA code: `kcgtotl1rd00`). Since both series are indexes and, therefore, measure capital accumulation they do not contain information about capital level. Thus, we take the nominal value of net R&D capital stock to get an estimate of the real capital stock. Namely, we use current cost net R&D stock in 2012 in private (BEA code: `k1ntotl1rd00`) and public sector (BEA code: `k1gtotl1rd00`). Next, it is assumed that public and private R&D capital are perfect substitutes, i.e., the elasticity of substitution between these inputs tends to ∞, and we simply sum estimated aggregates.

In the second approach, we use the perpetual inventory method. The capital stock (K_t) is the

[2] The available data (in constant dollars) starts in 1999. This time span is too short to analyze the long-run patterns in R&D productivity.

sum of the capital stock in previous period reduced by deprecation and investment at period t:

$$\mathcal{K}_t = (1-\delta)\,\mathcal{K}_{t-1} + I_t^{rd} \tag{A.1}$$

where δ is the deprecation rate of R&D capital and I_t is the real investment in R&D. A key problem in calculating capital based on the PIM (A.1) formula is the initial condition problem. We follow OECD (2009) and apply the following formula:

$$\mathcal{K}_0 = I_0^{rd}/(g+\delta) \tag{A.2}$$

where g is the long-run (geometric) growth rate of R&D investment.

While the g element can be easily calculated from historical data there is a lot of uncertainty about the deprecation rate of the R&D capital. The standard choice in the literature is to fix deprecation rate at 15% (Venturini, 2012). More recently, there are several studies that provide empirical evidence that suggest a higher depreciation rate. Bernstein and Mamuneas (2006) find that deprecation rate is above 15% while Li and Hall (2019) document that the depreciation rate of R&D capital is even above 30%. In the 2019 KLEMS vintage, the deprecation rate for the R&D assets is fixed at 20%.[3]

In addition, the BEA publish historical series on deprecation of R&D capital. As previously, deprecation of R&D capital is also available separately for private and public sector. According to the BEA estimates, the implied deprecation rate is slightly above consensual value of 15%. However, the BEA estimates suggest that the deprecation rate has not been constant over time (figure A.7). Before the WWII substantial short-run variation in the depreciation rate for both public and private R&D capital can be observed. This is due to approximation error related to the available BEA statistics. Namely, the BEA publish data expressed in billions of dollars and rounded to one digit. Therefore, in extreme case, i.e., for public capital, there is no depreciation of R&D capital in the 1920s because the reported value of depreciation is zero. Abstracting from this period, the implied deprecation rate has been stable since the WWII to the late 1950s. After that, there is unquestionable declining trend in deprecation rate. In our baseline setting, we use standard in the literature assumption that annual depreciation rate equals 15%. However, based on above discussion, we will carefully document a sensitivity of this choice.

To apply the PIM method we proceed as follows. Since there is no measures of real R&D investment expressed in chained dollars we estimate the series of interest based on available

[3] See https://euklems.eu/wp-content/uploads/2019/10/Methodology.pdf.

series, i.e, nominal R&D investment data as well as price indexes. For private sector, we divide nominal R&D investment (BEA code: `Y006RC`) by price index of this asset (BEA code: `Y006RG`). The same strategy is applied for public sector (BEA codes `Y057RC` and `Y057RG`, respectively). In addition, we also consider the following components of public investment: federal non-defense (BEA codes: Y069RC and Y069RG), defense (BEA codes: `Y076RC` and `Y076RG`) and state and local (BEA codes: `Y073RC` and `Y073RG`).

Based on the constructed series we can formulate the following stylized facts:

- The R&D capital stock has a unit root.[4] Non-stationarity of R&D capital is quite an intuitive feature as it can be expected that in the economy there is some accumulation of R&D capital. This implies that R&D capital should be rather an upward trending than a stationary variable. Technically speaking (see equation (A.1)), R&D capital would be stationary if investment in new capital (I_t) equals over the time depreciated capital stock. This case seems to be unrealistic.

- The dynamics of accumulation in aggregate R&D assets is complex. There are several time series features that can be simultaneously documented.

  - There is a downward (almost linear) trend in growth rate of R&D capital.

  - Even after differentiating the R&D stock are highly persistent. This is suggested by high persistence estimates obtained for the $AR(1)$ model.

  - There is a visible structural break in the R&D capital accumulation. Since the 1970s the growth rate of the R&D capital has dropped permanently. This can be observed for both the FAT and PIM based series. In addition, the above visual investigation is confirmed by a broad range of statistical tests.

  - The role of short-run fluctuations has been declining over the time. To evaluate the role of business cycles and medium-term variation we use a band pass filter proposed by Christiano and Fitzgerald (2003) and decompose all fluctuations in three groups: short-run/business cycles (frequency higher than 8 years), medium-term swings (from 8 to 50 years) and long-run oscillation and long-run trend (frequency below 50 years). Visual inspection of spectral decomposition suggests that the role of business cycles in shaping the R&D accumulation was significant only before the 1970s. Since the beginning of the 1970s the R&D accumulation is mostly driven by long-run trend and medium-run swings. Substantial magnitude of the long-run and medium-run fluctuation is

[4] We applied a battery of unit root and unit root under structural break tests. These are available on request.

consistent with high persistence that can be found for annual growth rates of R&D assets.

- The accumulation of R&D capital has been faster on average in comparison with total assets.
- The share of public/private assets in the total R&D capital has not been stable over the time. The following periods can be identified (see figure A.12)
  - Sudden and substantial rise in public R&D capital during the WWII due to increasing role of defense R&D.
  - Slight upward trend in share of public R&D assets in total R&D assets due to rising role of defense sector as well as space programme.
  - Diminishing role of public assets in total R&D capital since the beginning of the 1970s.
- The properties of the PIM-based series of the R&D capital are slightly sensitive to a choice of (i) deprecation rate, and (ii) initial period.

  The characteristics of the PIM-based series depend on the depreciation rate as well as initial year (A.1). To check sensitivity of properties of the PIM based series of R&D capital we calculate the counterfactual PIM series (i) using various values of depreciation rate, (ii) truncating recursively available sample. To scrutinize an effect of these changes we calculate long-run averages. In addition, we consider two measure of co-movement with the FAT-based measures of the R&D capital. First, the correlation between annual growth rates which measures short-run co-movement. Second, we employ long-run approach which is related to testing the cointegration.

  The long-run properties of the PIM-based series are not extremely sensitive to a choice of initial year and deprecation rate. Figure A.8 illustrates a dependence of geometric growth rate and average annual growth rate of R&D capital on deprecation rate and initial period. There is a natural trade-off between assumed deprecation rate of the R&D capital and its long-run growth rate. For a higher deprecation rate more investment is required to replaced obsolete R&D capital and this implies slower R&D capital accumulation. However, this effect is not substantial. For extreme values of the depreciation rate, i.e., 0% and 40%, the average annul growth rate as well as geometric growth rate do not differ so much and range from 6% to 7% per annum.

  Moreover, the long-run average rate of accumulation of R&D capital depends substantially on a choice of initial period but this relationship is consistent with the long-run slowdown in R&D accumulation. Both the PIM-based series and the FAT-based measure exhibit a visible decline in growth rates of available R&D stock (see Figure **??**, right panel). This fact is consistent with previous evidence in favor of occurrence of structural break.

Finally, we look at co-movement between the FAT-based measure and various PIM-based proxies that base on different values of δ. At the first sight, there is extremely high positive short-run correlation between considered series (Figures A.9 and A.10). In particular a choice of time invariant deprecation rate has no impact on the analyzed degree of co-movement as the lowest estimated correlation coefficient is above 0.9. The short-run correlation is slightly lower for detected previously structural break (in the late 1960s/ early 1970s) nut it is still significantly positive. The analysis of potential impact of our PIM assumptions on the long-run co-movement with the FAT-based series is more puzzling. In our baseline case, i.e., $\delta = 0.15$, there is no strong evidence in favor of the cointegration between the analyzed time series. Abstracting for the low power of unit root and cointegration test, the reason for that is that has been structural breaks in the analyzed series and its effect on the DGP (data generating process) could be not proportional.

– The constructed series of the R&D capital are comparable to measures in other databases. We use two additional data sources that offer data on the US R&D capital. First, we use capital the R&D capital stock from KLEMS database (van Ark and Jäger, 2017). Second, we use an index of R&D capital services from the Multifactor Productivity (MFP) database provided by OECD.

  All series are shown in Figure A.3. It is straightforward to observe that short-run co-movement between those series is high. Moreover, the average growth rate is very similar among the considered sources.

– Finally, the share of the R&D assets in total capital stock has been systematically rising from the 1920s to 1980s and it has been roughly stable since the beginning of the 1980s. All in all, the average share of the R&D capital in nonresidential (total) available assets has been fluctuated around 8% (5%) since the beginning of the 1980s. This empirical pattern is mostly determined by a rising role of R&D intensity in private sector. The share of the R&D assets in private capital stock has been systematically risen since the 1920s. At the same time, the share of R&D in public assets exhibits hump-shaped trajectory, reaching the maximum in the 1980s.

A.3 R&D Labor

Estimation of the Idea production function requires data on labor engaged in the research and development process. At the conceptual level and in line with the definition from the Frascati Manual (OECD, 2015) it refers to employees who undertake creative work that is aimed at general increase in an existing stock of knowledge.

In practice, an application of the above definition requires very detailed information about tasks that are related to R&D activity. However, according to the best of our knowledge such data are not available. As a result, R&D activity cannot be measured directly. Therefore, we will use two strategies.

First, we take advantage of publicly available data on R&D employment. Although statistical offices (Eurostat or OECD) publish estimates on R&D activity their availability is strongly limited. Namely, the Eurostat/OECD series starts in 1981. To overcome this problem we use older vintages in order to extrapolate existing series. Before 1981 we use data collected within the IRIS (Industrial Research and Development Information System) program conducted by the NSF (the National Science Foundation). Moreover, based on historical data from Jones (1995) it is possible to get extrapolated observations earlier, i.e., in 1950s.

In our second approach, we estimates the labor input in R&D activity using microdata which contains information about structure of occupations. An ideal strategy is to use detailed data on skills/abilities-content in occupations and merge them with structure of labor force. The most important problem with this approach is that, according to our best knowledge, there is no longitudinal survey on research-intensity among occupations. For instance, the O-NET data offers estimates on skill and abilities intensity but there is no direct measure of research intensity and the time span of this dataset is quite short since this survey started in 1998.

Thus, in our empirical part, we use IPUMS CPS data (Ruggles et al., 2019). This database offers harmonized micro data, namely the Current Population Survey (CPS), i.e., the monthly U.S. labor force survey. Based on the conceptual definition of R&D personal or the S&E groups we can identify the following occupational groups whose work could be classified as R&D activity[5]: **Scientists**; **Mathematical and computer occupations**; **Engineers**; **Technicians**; **Social scientists** and **Health professionals** (see the main text for precise definitions).

Based on above available classification we define two aggregate of the R&D labor. In our baseline definition, we include scientists, mathematical and computer occupations and engineers.

[5] In practice, we try to match the Eurostat definition. According to human resources in science and technology approach scientists and engineers (S&E) are workers who conduct research, improve or develop concepts, theories and operational methods and/or apply scientific knowledge relating to fields. This definition can be covered by following group occupation (according to ISCO-08 classification): Science and engineering professionals (21), Health professional (22) and Information and communications technology professionals (25).

In addition, we consider a broader definition that includes also technicians, social scientist and health professionals.

The following set of stylized facts can be formulated:

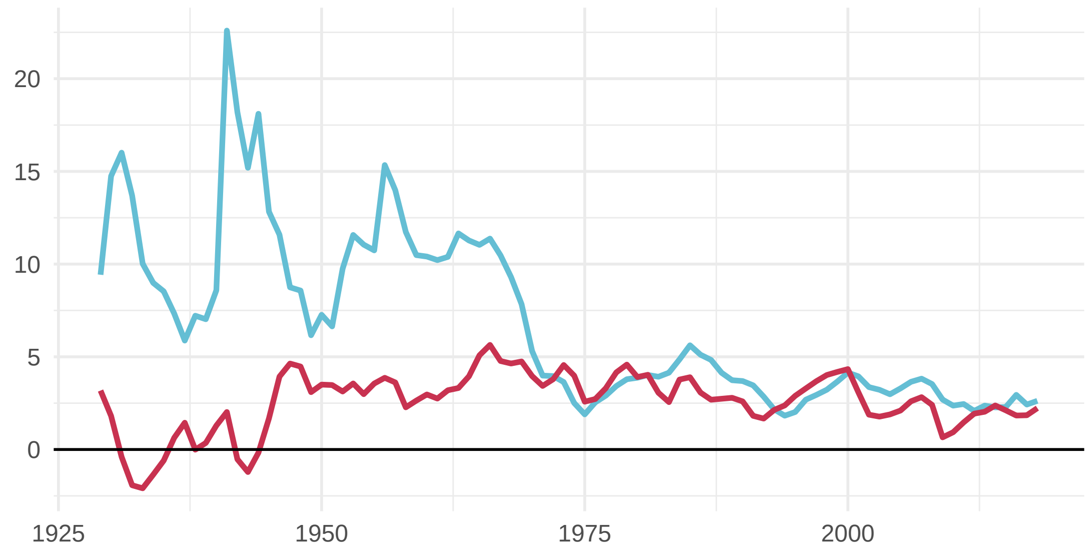
– Share of the R&D related workers is total employment is rising over the time. This is consistent with the previous empirical evidence in the literature (Jones, 1995; Ha and Howitt, 2007).

– There are substantial differences in the level of R&D employment. Even if we look at the series of the Scientists&Engineers the share of this group in total employment according to data provided by (Jones, 1995) is almost two times higher than the share which is estimated based on NSF/Eurostat data. By definition, this difference would be higher if we compare with the (IPSUM-based) share broader group of occupations.

– Nevertheless, all proxies of the R&D employment suggest almost identical upward tendency. According to the merged series on Scientists&Engineers and the IPUMS-based group of scientists the share of R&D employment in total employment rose by around 80% between 1968 and 2017. For the baseline IPUMS-based definition this increase has been even larger and exceeded 100%.

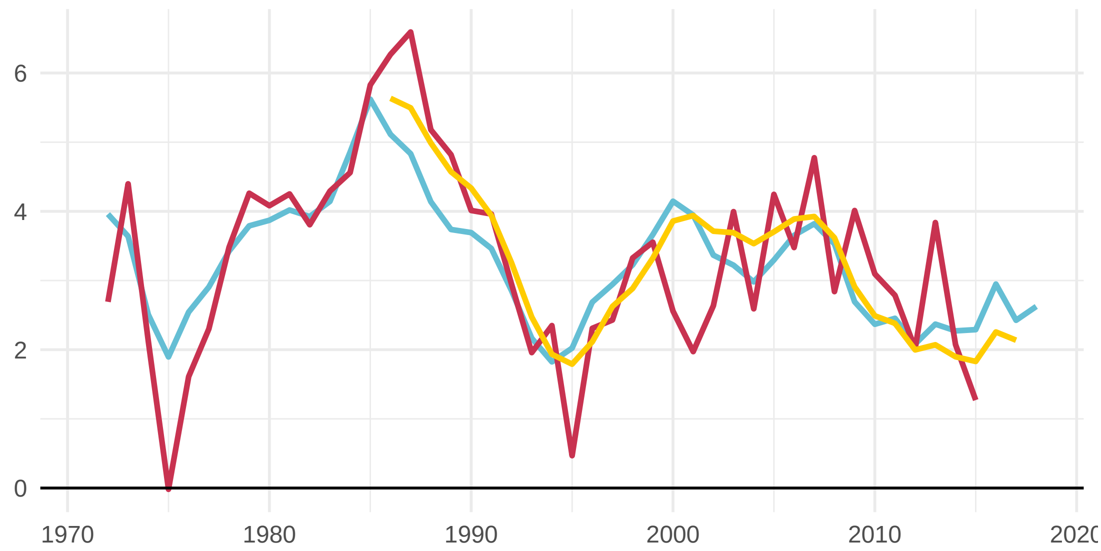
– Detailed decomposition of the IPUMS-based measures illustrates key measurement problem. An on-going technical change has created demand for new occupations that are closely related to the new technologies. This is mostly observable for computer-related occupations. In the late 1960s this occupational group was almost absent on the labor market while in the late 2000s their share (together with mathematical occupation) was around 4%.

Figure A.2: Total R&D capital (annual growth rate) and Nonresidential Private Fixed Assets



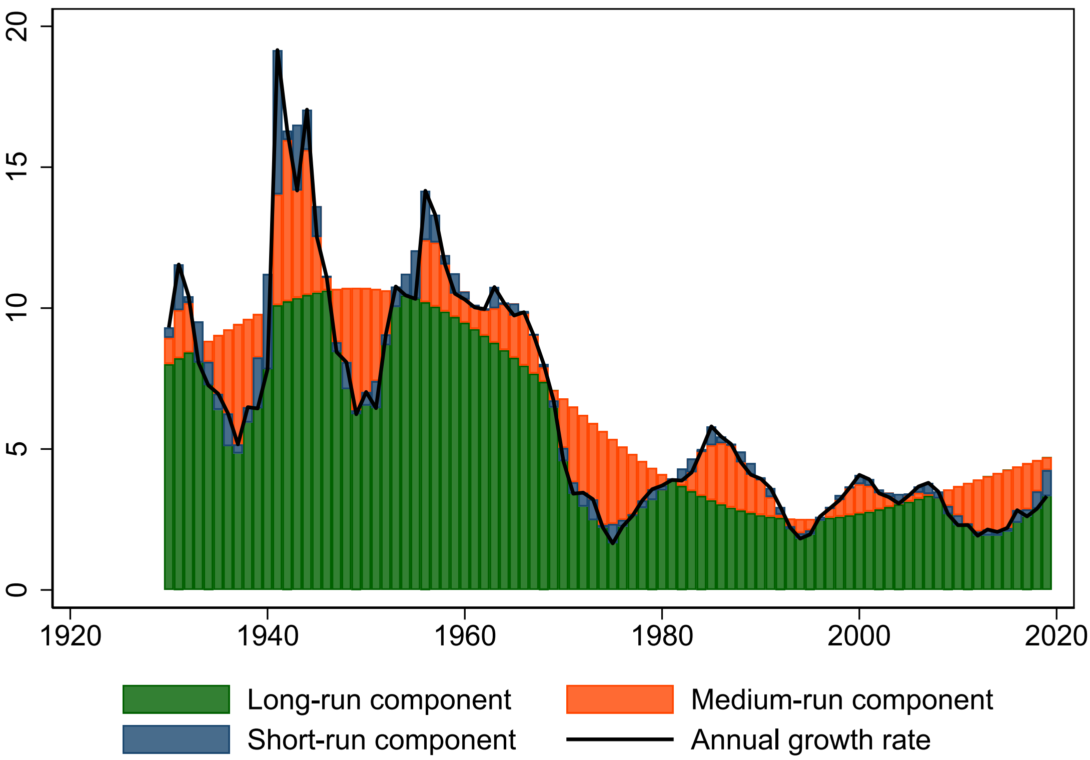
Notes: red color denotes nonresidential private capital while blue line stands for total R&D capital.

Figure A.3: The comparison of the R&D stock with other data sources (annual growth rate)



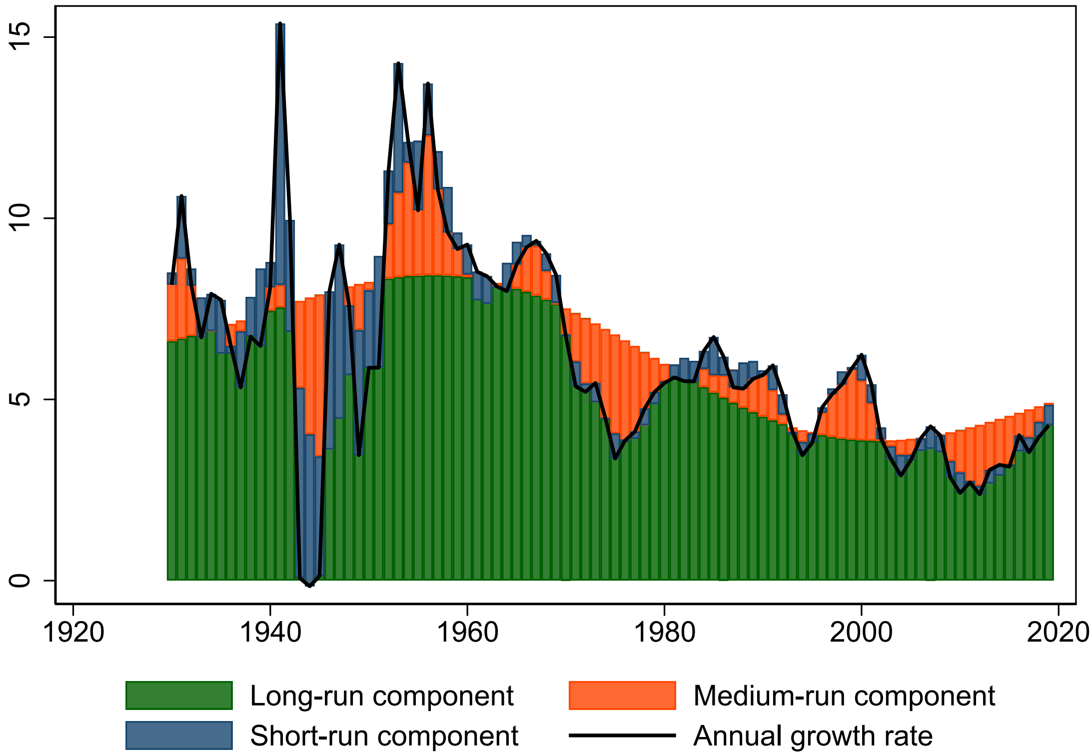
Notes: blue line is the BEA-based estimates of R&D stock, red color denotes the KLEMS estimate and yellow color is the OECD estimate of the R&D capital.

Figure A.4: Spectral decomposition of the total R&D capital (annual growth rate)



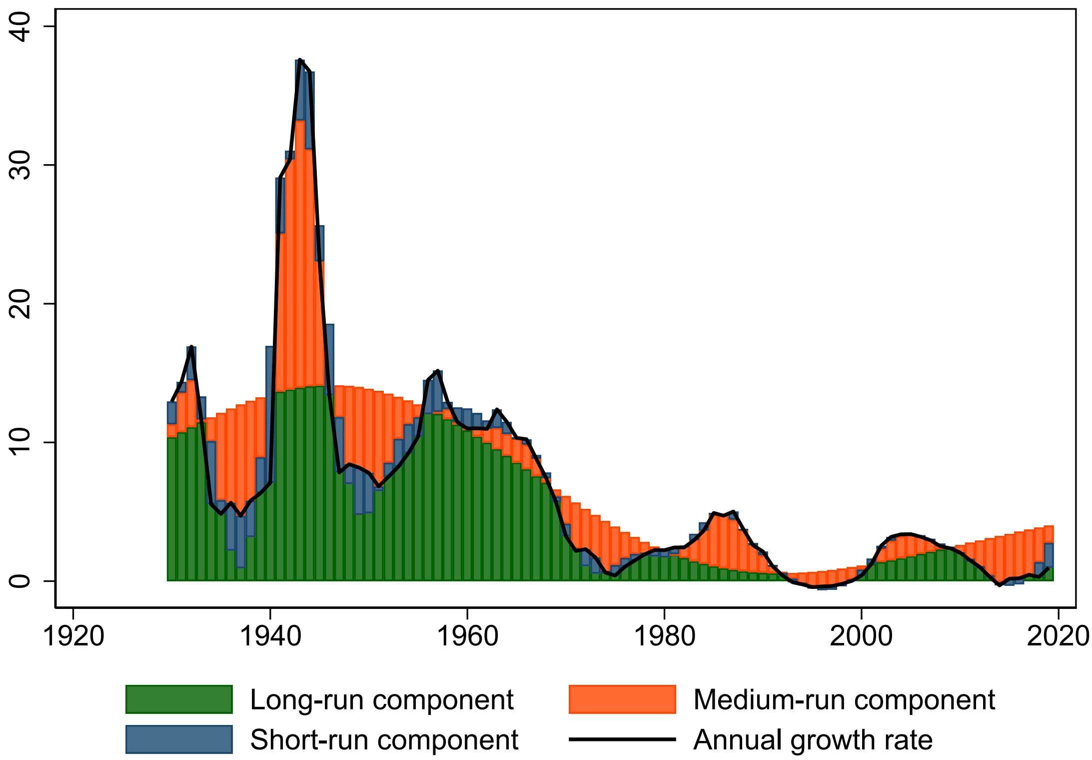
Notes: See main text for description.

FIGURE A.5: Spectral decomposition of the private R&D capital (annual growth rate)



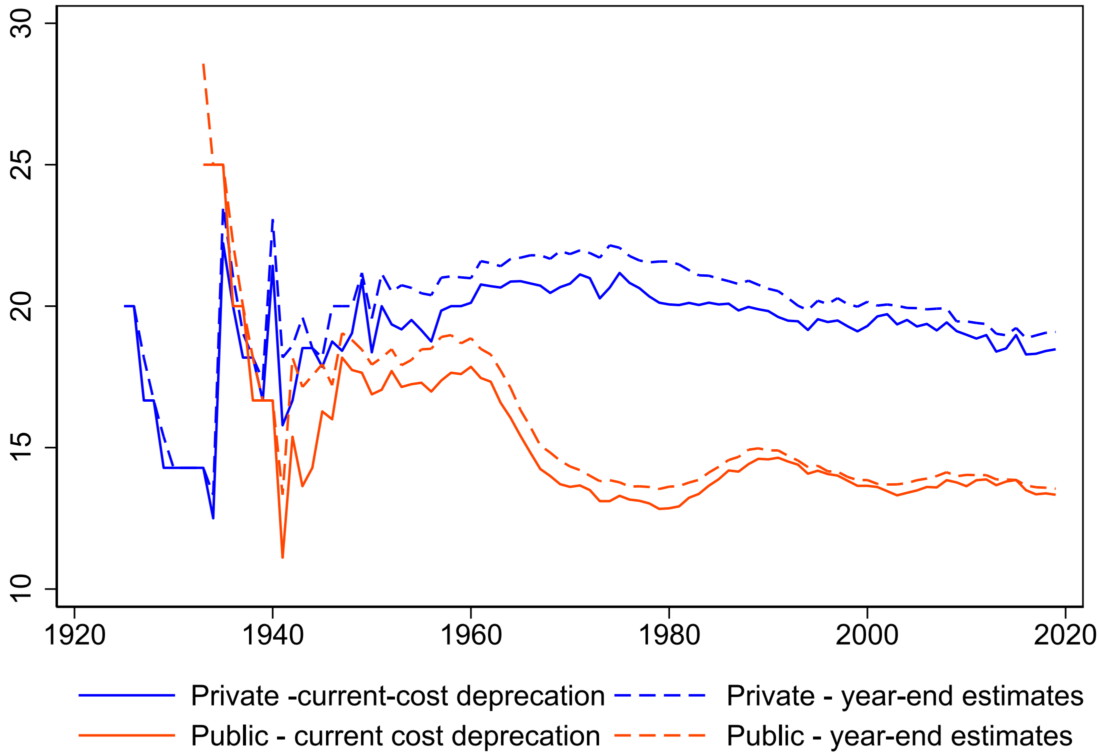
Notes: See main text for description.

Figure A.6: Spectral decomposition of the public R&D capital (annual growth rate)



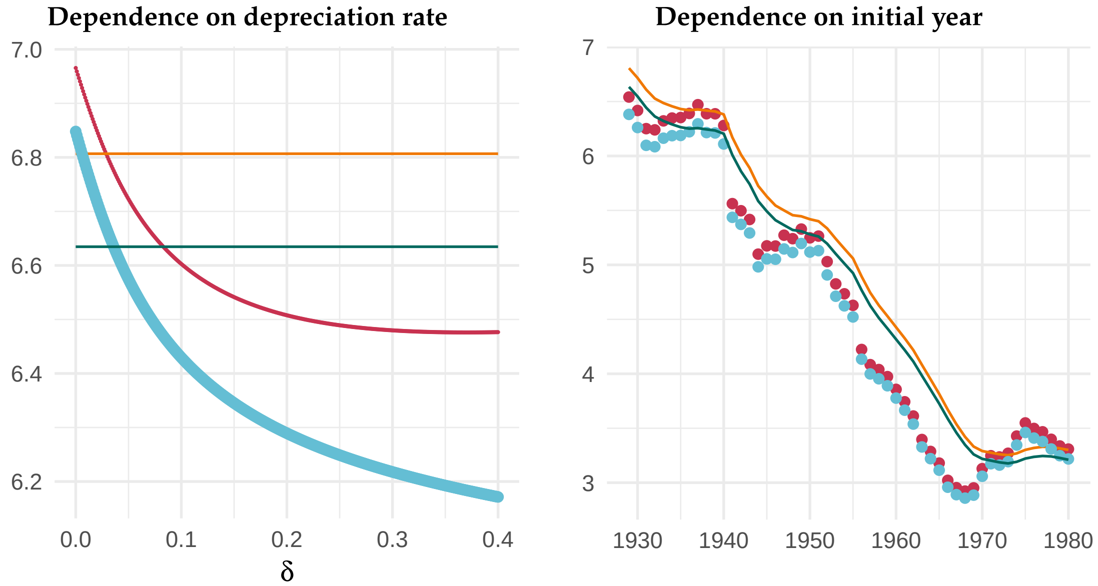
Notes: See main text for description.

Figure A.7: Implied deprecation rate of R&D capital based on the BEA data



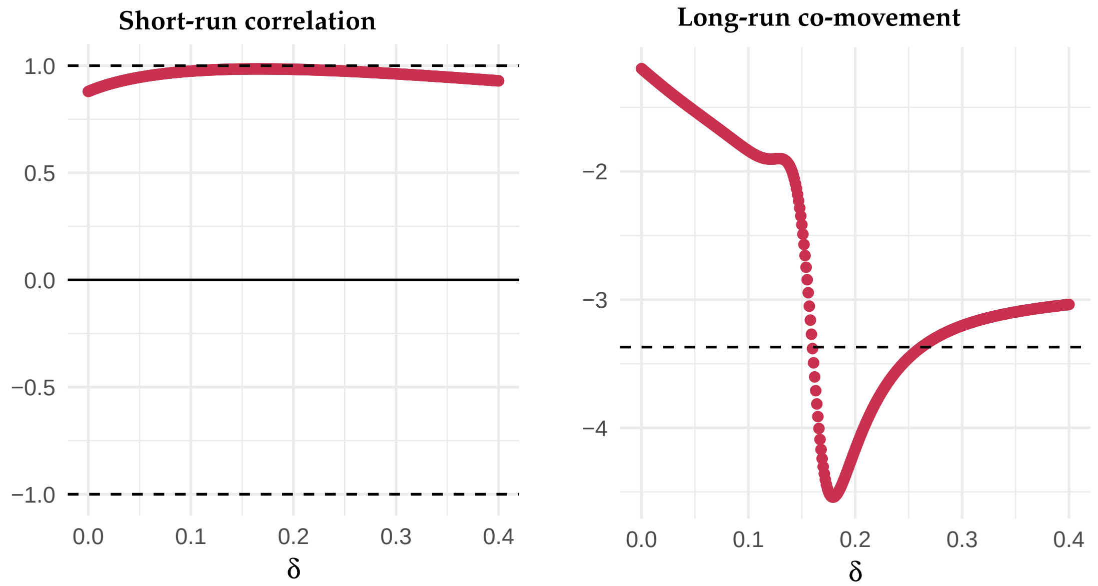
Notes: Implied depreciation rates for public and private R&D capital.

FIGURE A.8: Sensitivity analysis of the PIM-based series to a value of the depreciation rate (δ) and a choice of initial year



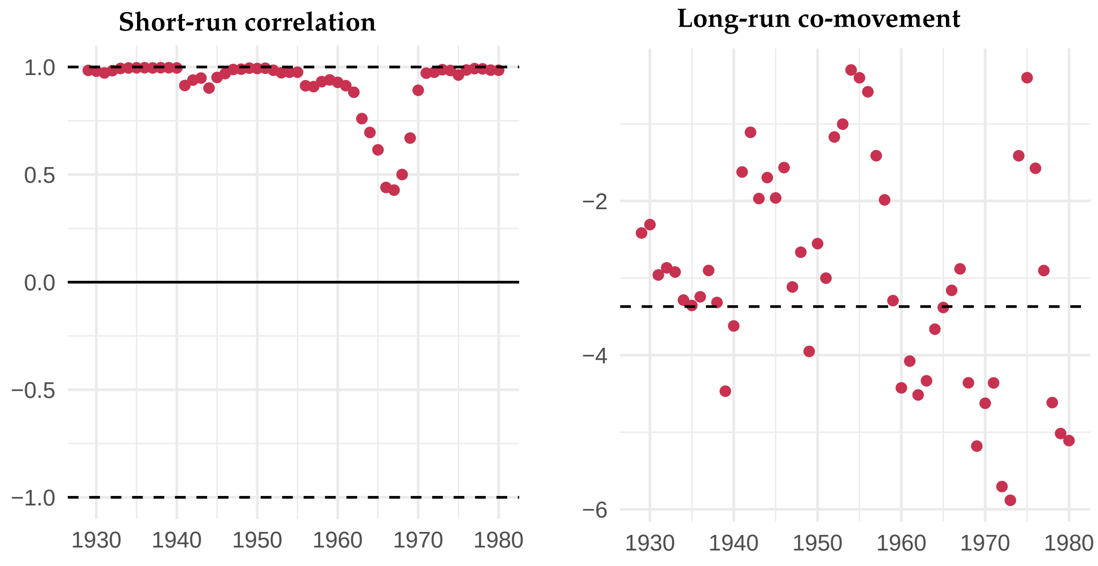
Note: blue color denotes average growth rate of the PIM-based measure, red color stands for geometric growth rate of the PIM-based measure, green color represents average average growth rate of FAT-based measure and orange color stands for the FAT-based geometric growth rate of FAT-based measure..

FIGURE A.9: Sensitivity analysis of the PIM-based series to a value of the depreciation rate (δ)



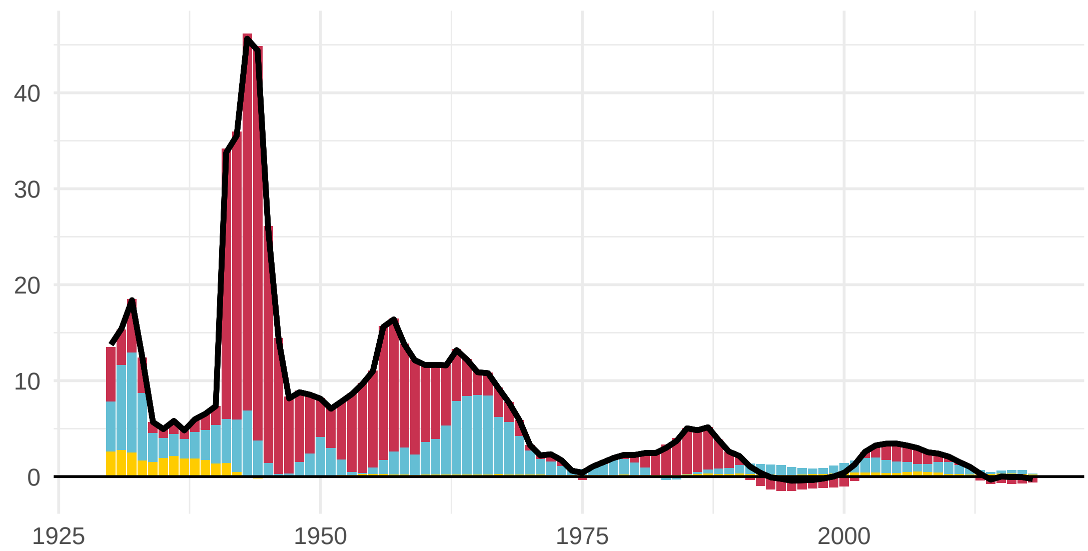
Notes: **Short-run correlation** portrays correlation between annual growth rates of the FAT R&D capital and the PIM-based R&D capital. **Long-run co-movement** illustrates the ADF statistics from regression of the logged PIM-based R&D capital on the logged FAT-based R&D capital. The dashed horizontal line is the 5% critical value for testing co-integration.

FIGURE A.10: Sensitivity analysis of the PIM-based series to a choice of initial year



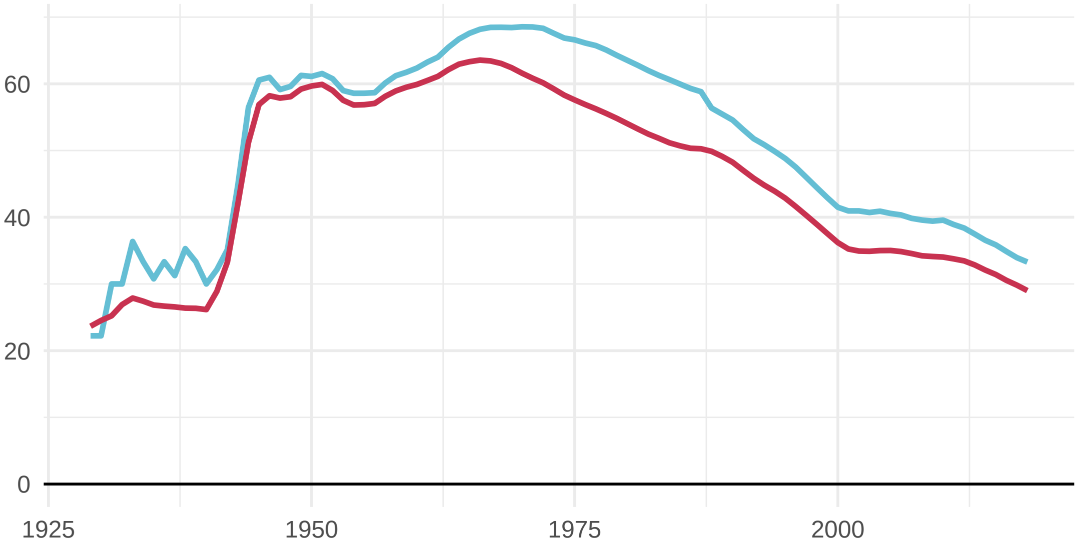
Notes: See figure A.9.

FIGURE A.11: Public R&D capital (annual growth rate) and its components



Notes: red color denotes defense R&D capital, blue color stands for non-defense federal R&D capital while yellow refers to state and local R&D capital.

Figure A.12: Share of public R&D capital in total R&D capital (in %)



Notes: red color denotes the share calculated using PIM-based series while blue color represents share obtained from (nominal) BEA series.

Figure A.13: Share of R&D assets in total and nonresidential capital stock (in %)

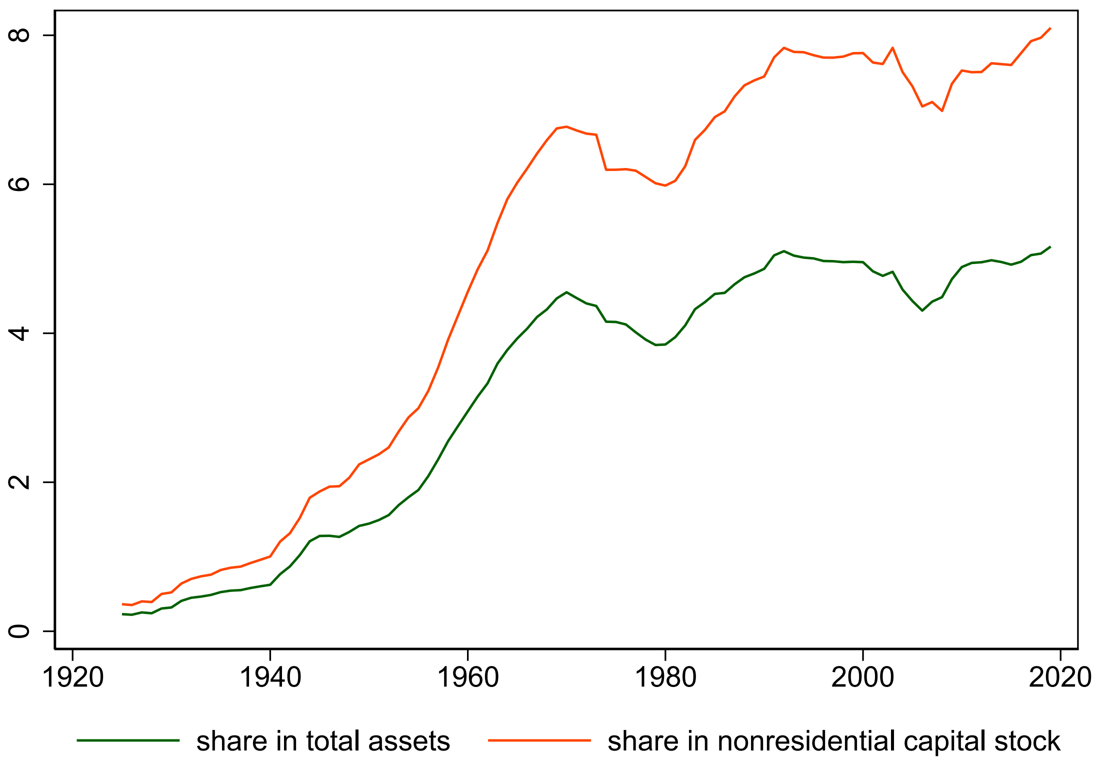
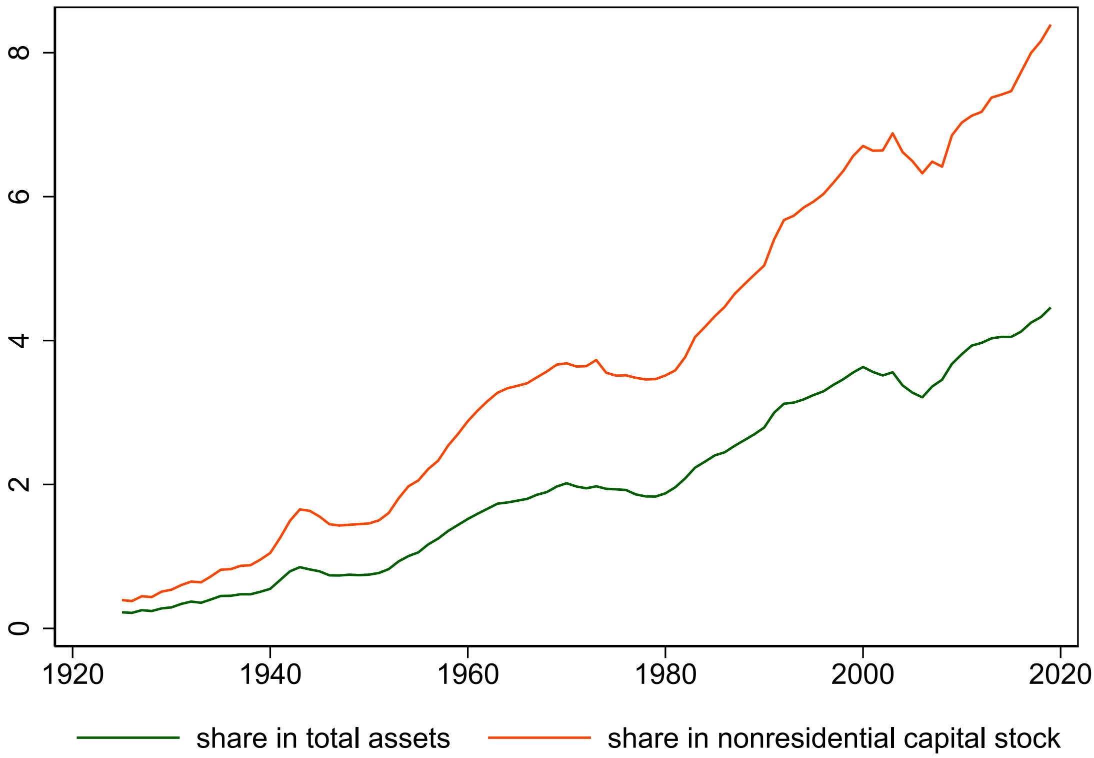
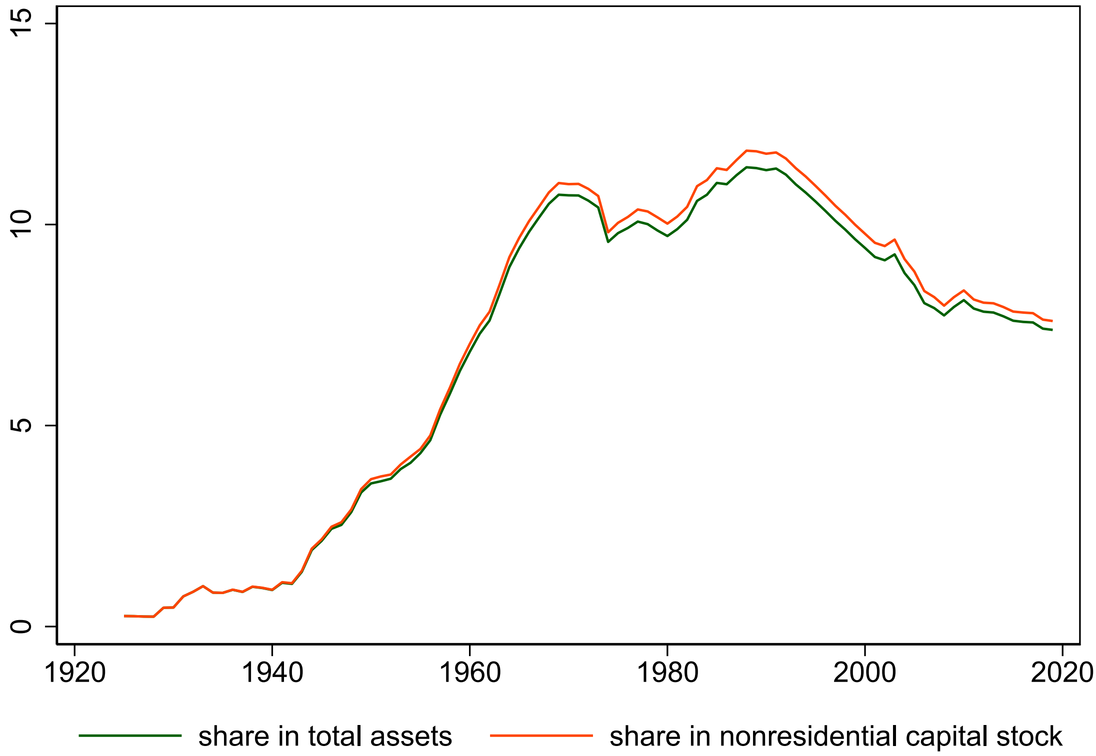


Figure A.14: Share of R&D assets in total private capital stock (in %)



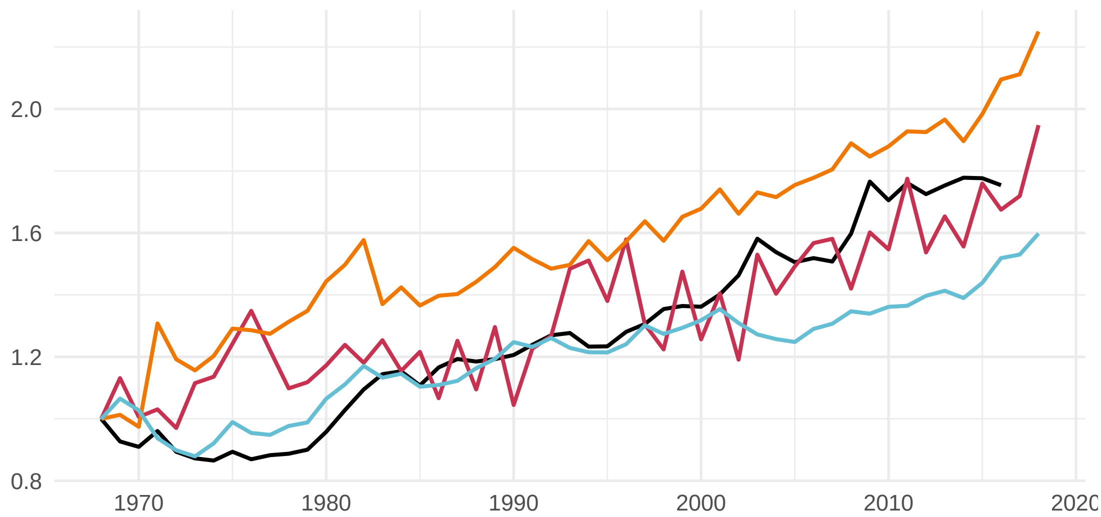
Notes: This figures shows private R&D assets by class.

Figure A.15: Share of R&D assets in total public capital stock (in %)



Notes: This figures shows public R&D assets by class.

Figure A.16: Share of the R&D employment (FTE, 1968=1)



Notes: The **black line is the merged (from various sources) series, red line is the IPUMS-based share of scientists, orange stands for the IPUMS based share of the R&D employees according to baseline definition while blue represents the IPUMS-based share of the R&D employees according to broader definition.**

Figure A.17: Share of the R&D related workers in total US employment and hours

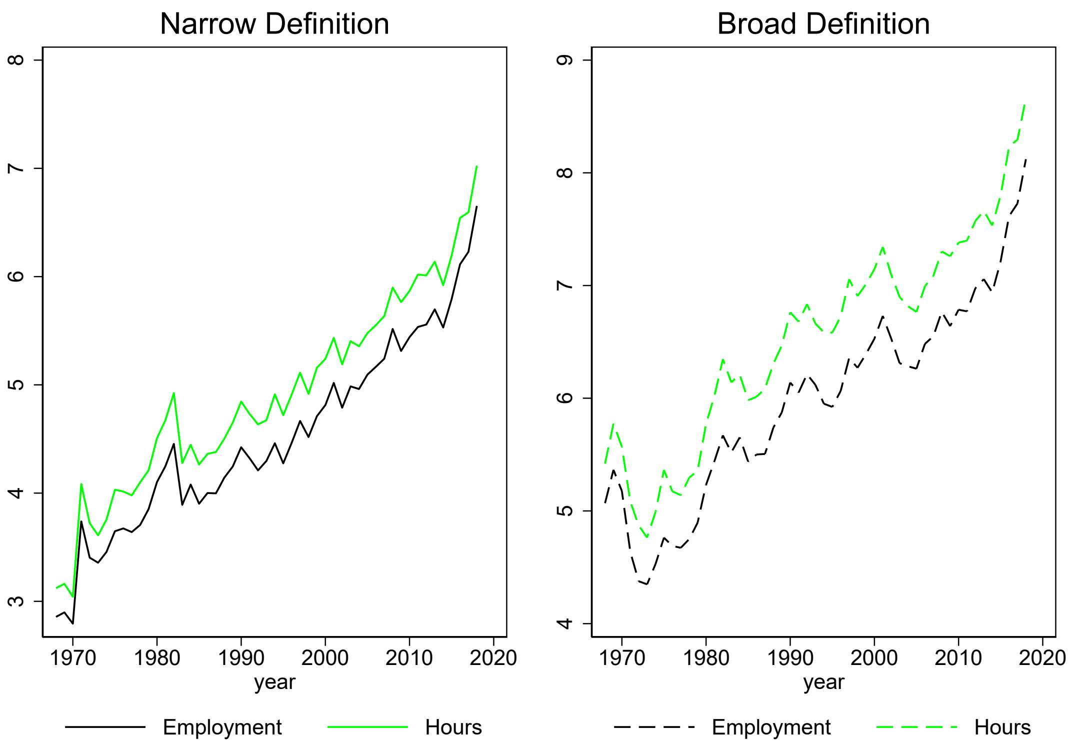
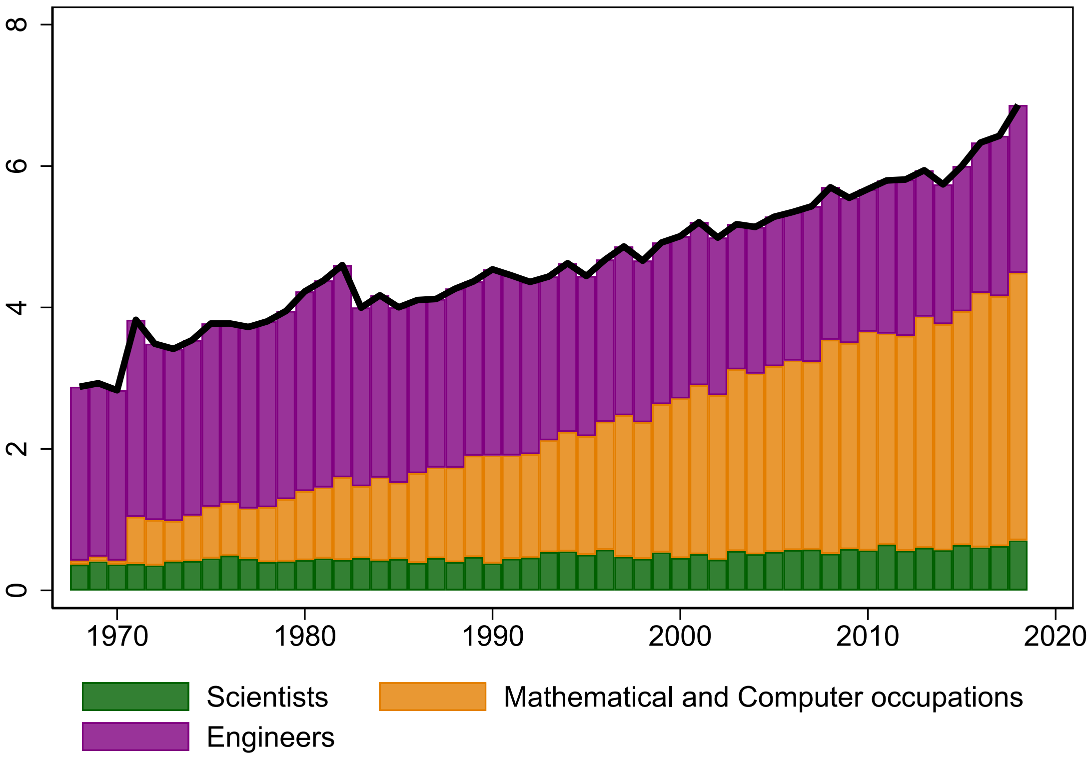
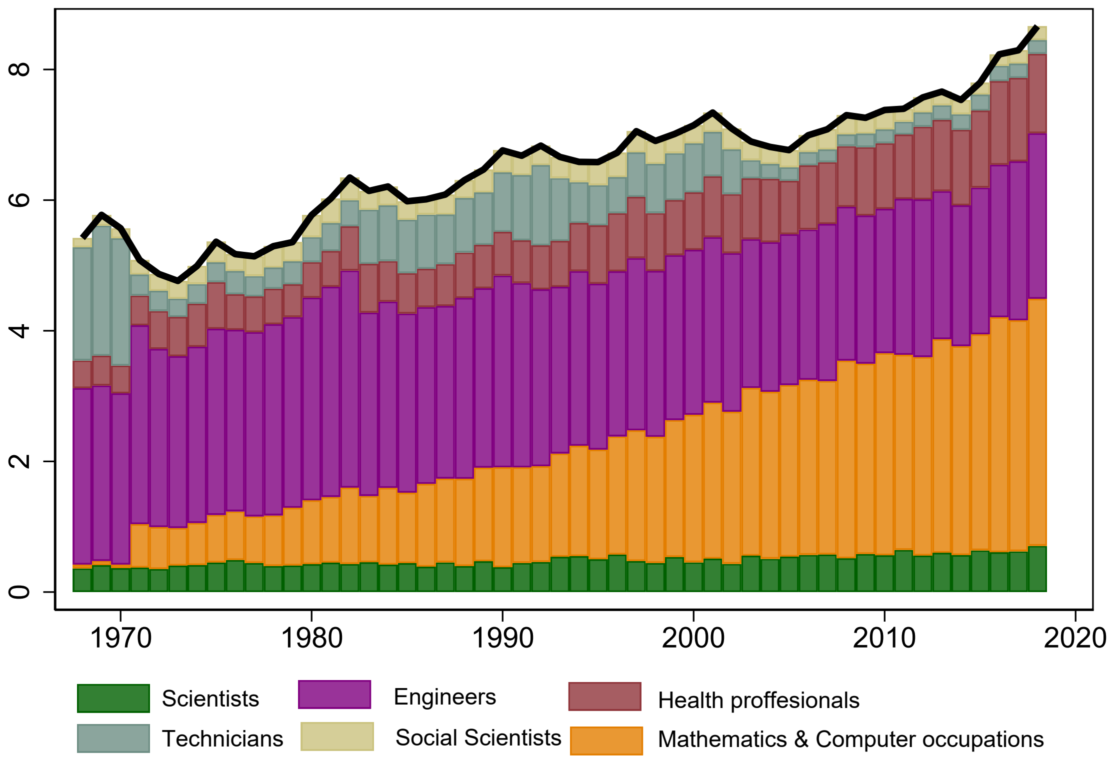


FIGURE A.18: Share of the R&D related occupations in aggregated hours (in %) – baseline definition



Notes: Narrower occupational definition of R&D occupations.

Figure A.19: Share of the R&D related occupations in aggregated hours (in %) – broader definition



Notes: Broader occupational definition of R&D occupations.

Figure A.20: Share of the R&D related occupation groups in total US employment and hours

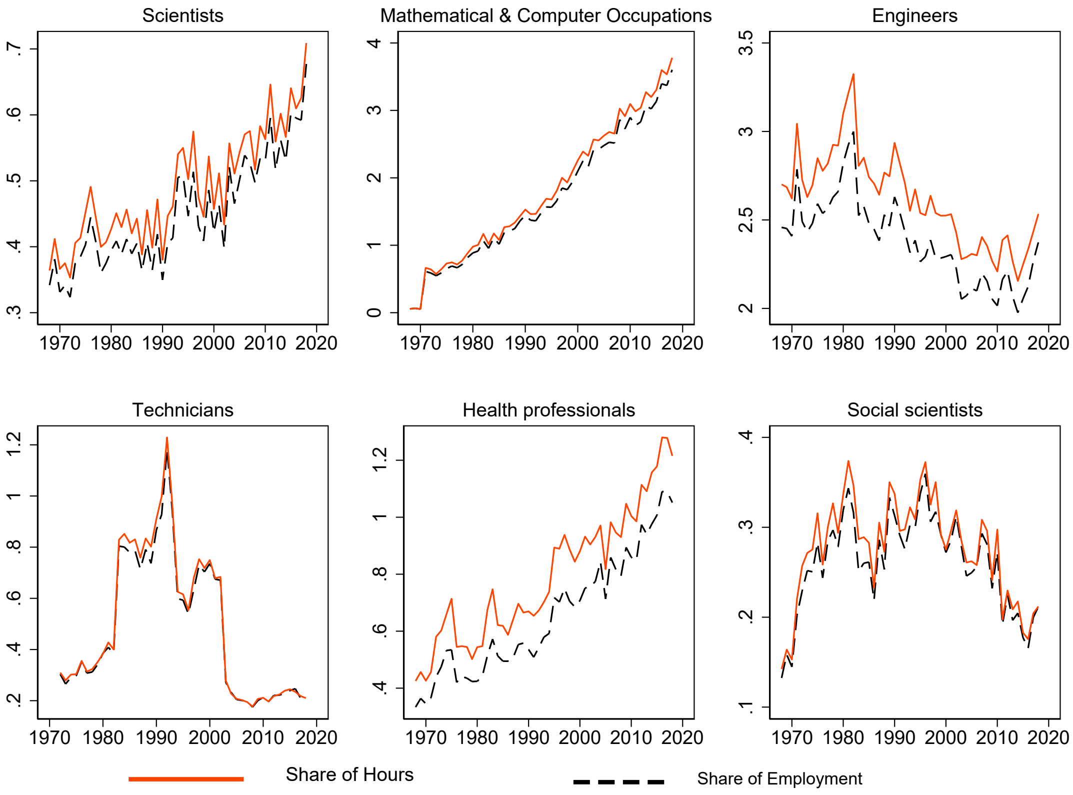
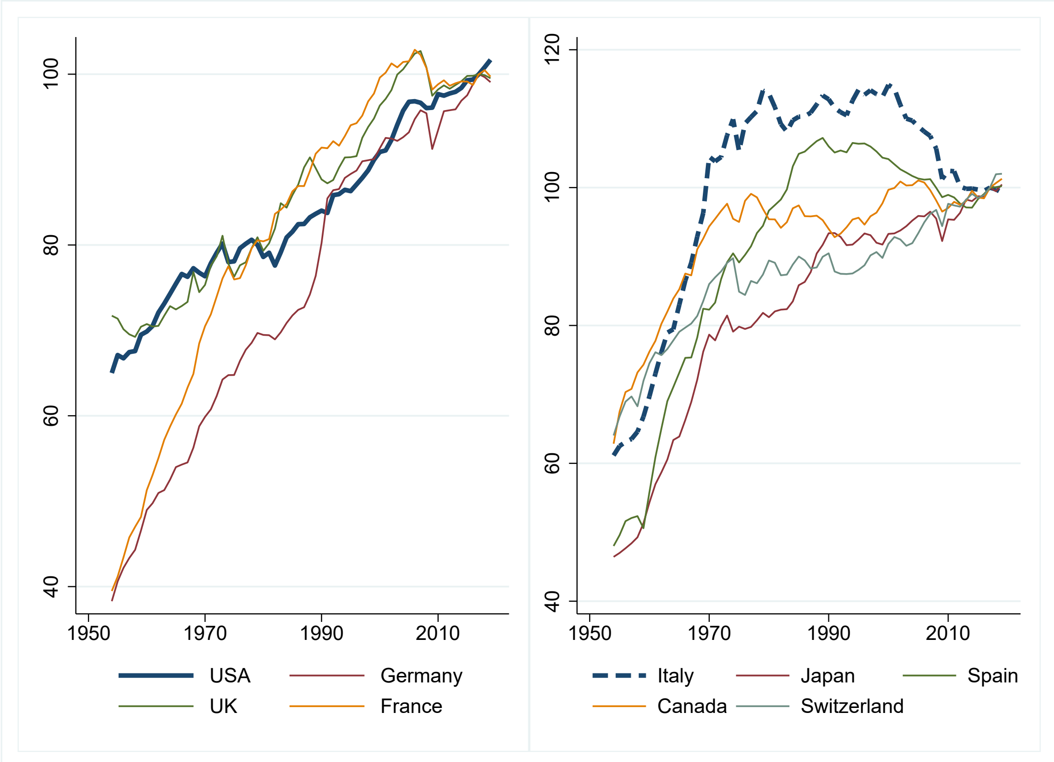


Figure A.21: Total Factor Productivity, 1954-2019: Constant National Prices, 2017=100 (G7 + Switzerland + Spain)



Source: Feenstra, Robert C., Robert Inklaar and Marcel P. Timmer (2015) "The Next Generation of the Penn World Table." *American Economic Review*, 105(10), 3150-3182. Indexed: 2017 = 100. The series were downloaded from FRED with the mnemonic `RTFPNAXXA632NRUG` where XX denotes the relevant country code, e.g. US.

B Unit Productivity Forms

B.1 Box Cox

Following Klump, McAdam and Willman (2007), we model time-varying technological progress terms using a Box-Cox transformation (specified in normalized form). This allows deterministic but time-varying technological progress terms where curvature or decay terms could be uncovered from the data in economically meaningful ways.

$$\Gamma_t^i = e^{g_t^i} \tag{B.1}$$
$$g_t^i = \gamma_i \times \left[\frac{\tilde{t}^{\lambda_i} - 1}{\lambda_i}\right] \times t_z, \tag{B.2}$$

The growth rate of technical change associated to factor i is therefore given by,

$$\gamma_t^i = \frac{dg_t^i}{dt} = \gamma_i \times \tilde{t}^{(\lambda_i - 1)} \tag{B.3}$$

where $\tilde{t} = t/t_z$ and curvature parameter $\lambda \in \mathbb{R}$ determines the shape of the technical progress function. Note, the re-scaling of γ and t by the fixed point value t_z in (B.2) allows us to interpret γ_i directly as the rates of i factor-specific unit productivity improvements at the fixed-point period ($t = t_z$).

For $\lambda = 1$, technical progress functions are the (textbook) linear specification; otherwise they are exponential ($\lambda \in (0,1)$), log-linear ($\lambda = 0$) or hyperbolic functions in time ($\lambda < 0$). If $\lambda > 1$ then technical progress is rapidly expanding; although essentially at odds with a BGP it is not impossible to observe such a pattern in a finite sample.

Asymptotically, function (B.2) would behave as follows in levels and growth rates, respectively:

$$\begin{aligned} &\lim_{t\to\infty} g_t^i \to \infty && \lambda_i \geq 0 \\ &\lim_{t\to\infty} g_t^i = -\tfrac{\gamma_i}{\lambda_i} t_z && \lambda_i < 0 \end{aligned} \tag{B.4}$$

$$\gamma_t^i = \frac{dg_t^i}{dt} = \gamma_i \times \tilde{t}^{\lambda_i - 1} \Rightarrow \begin{cases} \infty \text{ (as } t \to \infty) & \lambda_i > 1 \\ \gamma_i & \lambda_i = 1 \\ 0 & \lambda_i < 1 \end{cases} \tag{B.5}$$

B.2 Fourier

Our second case uses a trigonometric trajectory which is a special case of a Fourier expansion:[6]

$$\log \Gamma_t^j = \exp\left[(t - t_z)\left(\gamma_j + \kappa_j^{sin}\sin\left(\frac{2\pi\kappa t}{T}\right) + \kappa_j^{cos}\cos\left(\frac{2\pi\kappa t}{T}\right)\right)\right],$$

where $\pi = 3.14$ and $j = K, L$. Any possible structural breaks will be captured by the parameters $\kappa \in \mathbb{R}$ parameters, where $\kappa_j^{sin} = \kappa_j^{cos}$ retrieves the simple linear case. As regards the appropriate number of frequencies $\kappa \geq 1$ to include, we follow Ludlow and Enders (2000) who showed that a single frequency is invariably sufficient to approximate the Fourier expansion in the bulk of empirical applications.[7] Indeed higher values of k one might able capture rather low-frequency fluctuations in factor-biased technical change.

[6] See Christopoulos and León-Ledesma (2010) for a discussion of Fourier forms in economics.

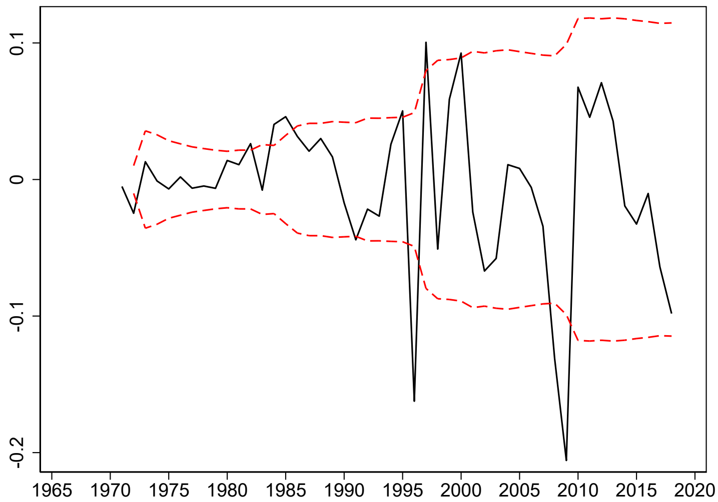
[7] Moreover, according to Becker et al. (2004) the Fourier expansion has more power to detect several smooth breaks of unknown form in the intercept than, say, the Bai and Perron (1998, 2003) multi-break tests.

C Stability Analysis

In this section, we perform a simple exploratory analysis of structural breaks on the patent growth series. We model it as a simple AR(1) process which should well capture its time series path.

We then estimate that form recursively over time and plot the persistence parameters and its associated standard errors. Values of the residuals outside of the standard error bands are indicative of structural breaks, large movements or cyclical swings. Looking at Figure C.1, we can see some suggestive evidence for structural instability for the periods around the mid 1980s, mid 1990s, around the Great recession.

Figure C.1: Recursive Residuals Stability Analysis for Patents



Notes: In this figure we derive the recursive residual (in black) plus/minus their two standard errors (in red dash) for an $AR(1)$ regression in $\Delta\widetilde{A}_t$. The interpretation of those recursive exercise being that residuals outside the standard error bands suggest instability in the parameters of the equation.

D Robustness

D.1 Alternative Measures of R&D Factors

As a robustness check we consider alternative empirical measures of R&D capital and R&D labor. The additional estimates for fixed and estimated η are presented in Table D.1. Qualitatively, all results replicate our previous preferred findings: the elasticity of substitution ξ is below unity; the average growth rate of R&D labor productivity is ranging from $0.1\% - 2.6\%$ per annum; there is evidence in favor of presence of a cyclical dynamic / multiple structural breaks in R&D capital productivity.

TABLE D.1: Robustness

	(1),(2)		(3),(4)		(5),(6)		(7),(8)	
	Baseline[†]		Narrow R&D Labor		Merged R&D Labor		Private R&D capital	
ξ	0.793^{***}	0.760^{***}	0.836^{***}	0.810^{***}	0.687^{***}	0.639^{***}	0.789^{***}	0.815^{***}
	(0.019)	(0.062)	(0.034)	(0.044)	(0.025)	(0.057)	(0.017)	(0.085)
$\gamma_{\mathcal{K}}$	-0.016^{***}	-0.013^{***}	-0.007	-0.006	-0.005^{***}	-0.005^{***}	-0.062^{***}	-0.074^{*}
	(0.003)	(0.004)	(0.007)	(0.005)	(0.002)	(0.001)	(0.005)	(0.042)
$\gamma_{\mathcal{R}}$	0.011^{***}	0.011^{***}	0.001	0.002	0.003^{**}	0.002^{**}	0.026^{***}	0.026^{***}
	(0.001)	(0.001)	(0.003)	(0.003)	(0.001)	(0.001)	(0.002)	(0.002)
$\gamma_{\mathcal{K}}^{sin}$	0.556^{***}	0.438^{***}	0.650^{***}	0.510^{***}	0.401^{***}	0.329^{***}	0.537^{***}	0.638^{*}
	(0.045)	(0.137)	(0.041)	(0.134)	(0.039)	(0.064)	(0.047)	(0.356)
$\gamma_{\mathcal{K}}^{cos}$	-0.427^{***}	-0.337^{***}	-0.406^{***}	-0.345^{***}	-0.398^{***}	-0.316^{***}	-0.548^{***}	-0.664^{*}
	(0.028)	(0.109)	(0.03)	(0.086)	(0.033)	(0.07)	(0.032)	(0.365)
η		0.418^{***}		0.405^{***}		0.426^{***}		0.280^{*}
		(0.121)		(0.1)		(0.104)		(0.149)
R&D Labor Productivity	Exp.	Exp.	Exp.	Exp.	Exp.	Exp.	Exp.	Exp.
R&D Capital Productivity	**F**	**F**	**F**	**F**	**F**	**F**	**F**	**F**
η	fixed	estimated	fixed	estimated	fixed	estimated	fixed	estimated
$\xi = 1$	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.031]
$\gamma_{\mathcal{R}} = \gamma_{\mathcal{K}}$	[0.000]	[0.000]	[0.388]	[0.332]	[0.000]	[0.000]	[0.000]	[0.017]
$\kappa_{cos}^{\mathcal{K}} = \kappa_{sin}^{\mathcal{K}} = 0$	[0.000]	[0.006]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.192]
res_3	[0.004]	[0.006]	[0.008]	[0.011]	[0.000]	[0.000]	[0.008]	[0.009]
res_4	[0.001]	[0.001]	[0.000]	[0.000]	[0.000]	[0.000]	[0.002]	[0.003]
ll	133.2	134.2	115.6	116.2	134.2	134.7	127.9	127.4
bic	-239	-237	-203.7	-201	-241.1	-238.2	-228.2	-223.4

Notes: The numbers in parentheses are standard errors, where the significance stars are to be read as $^{*}p < .1$, $^{**}p < .05$ and $^{***}p < .01$. Probability values are in brackets. In the diagnostic section of the table, res_3 and res_4 refer to ADF test of the units root null associated to the errors in equations (4) and the logged form of (3) and the p-values are obtained by bootstrapping distribution. Terms ll, bic denote, respectively, the Log Likelihood and Bayesian Information Criterion.

[†]: the baseline columns replicate the final two columns of Table 2.